Longevity Risk and the Econometric Analysis of Mortality Trends and Volatility

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Overview

- Longevity Trends for countries: Australia, England, Japan, Norway, USA
- Unit roots and Stochastic Trends
- Principal Components and Multiple Factors
- Cointegration and Common Stochastic Trends

Longevity Trends by Country

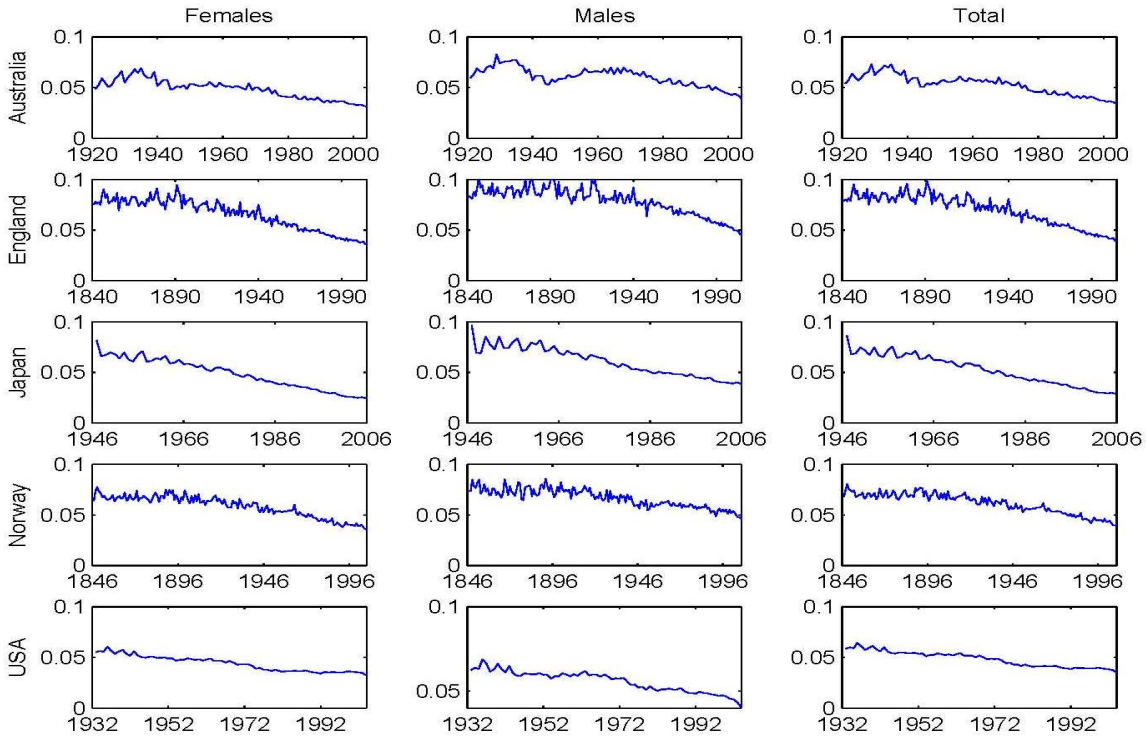
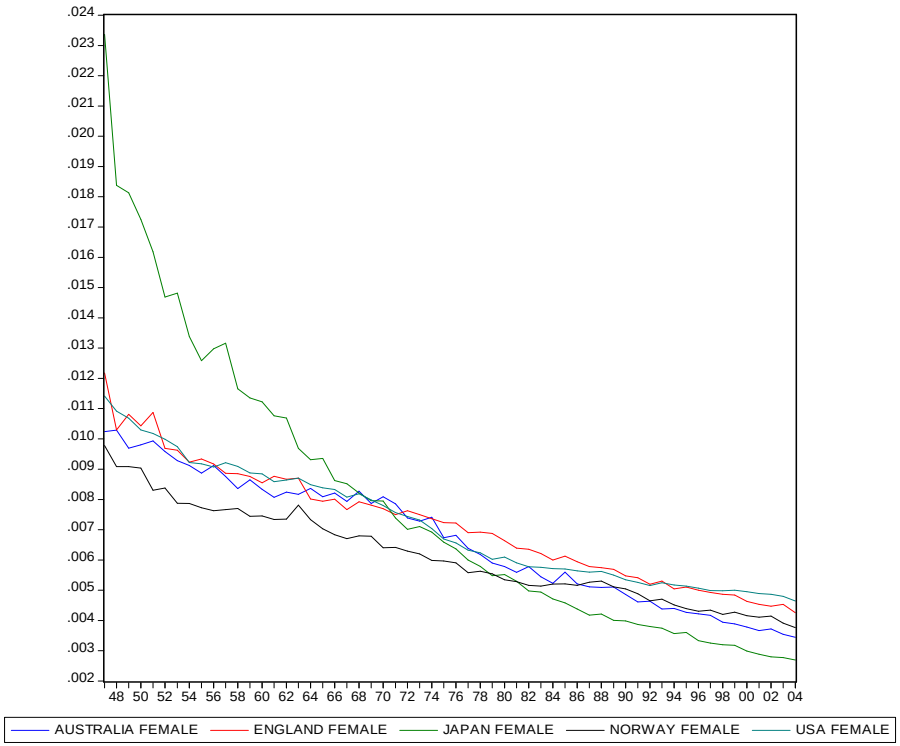


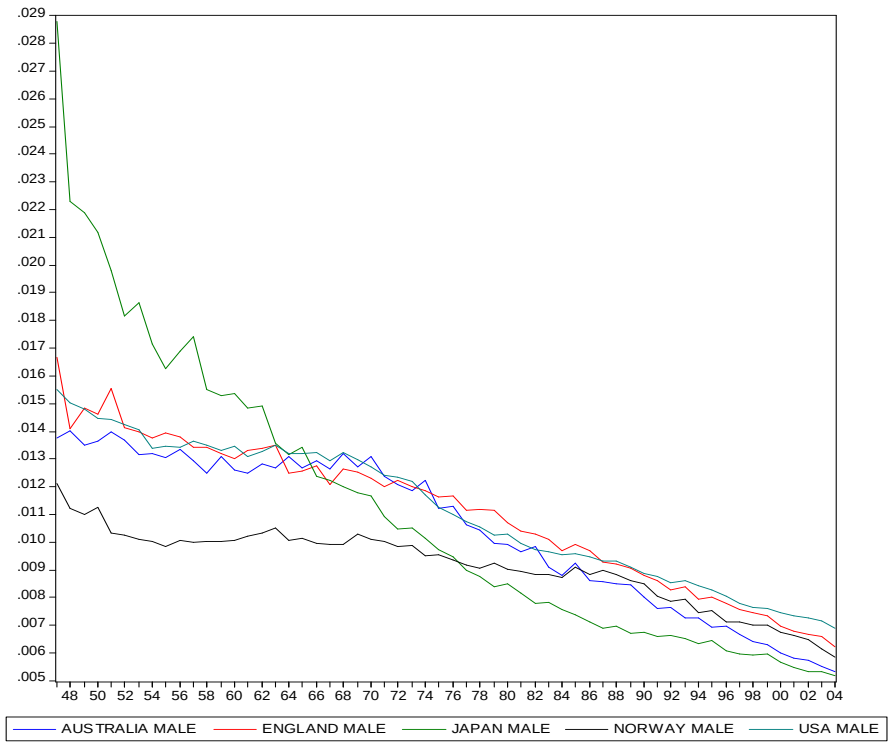
Figure: Average Death Rates for Five Countries in the Study

Longevity Trends by Country - Standardised Rates

Females



Males



Longevity Trends for Australia by Age

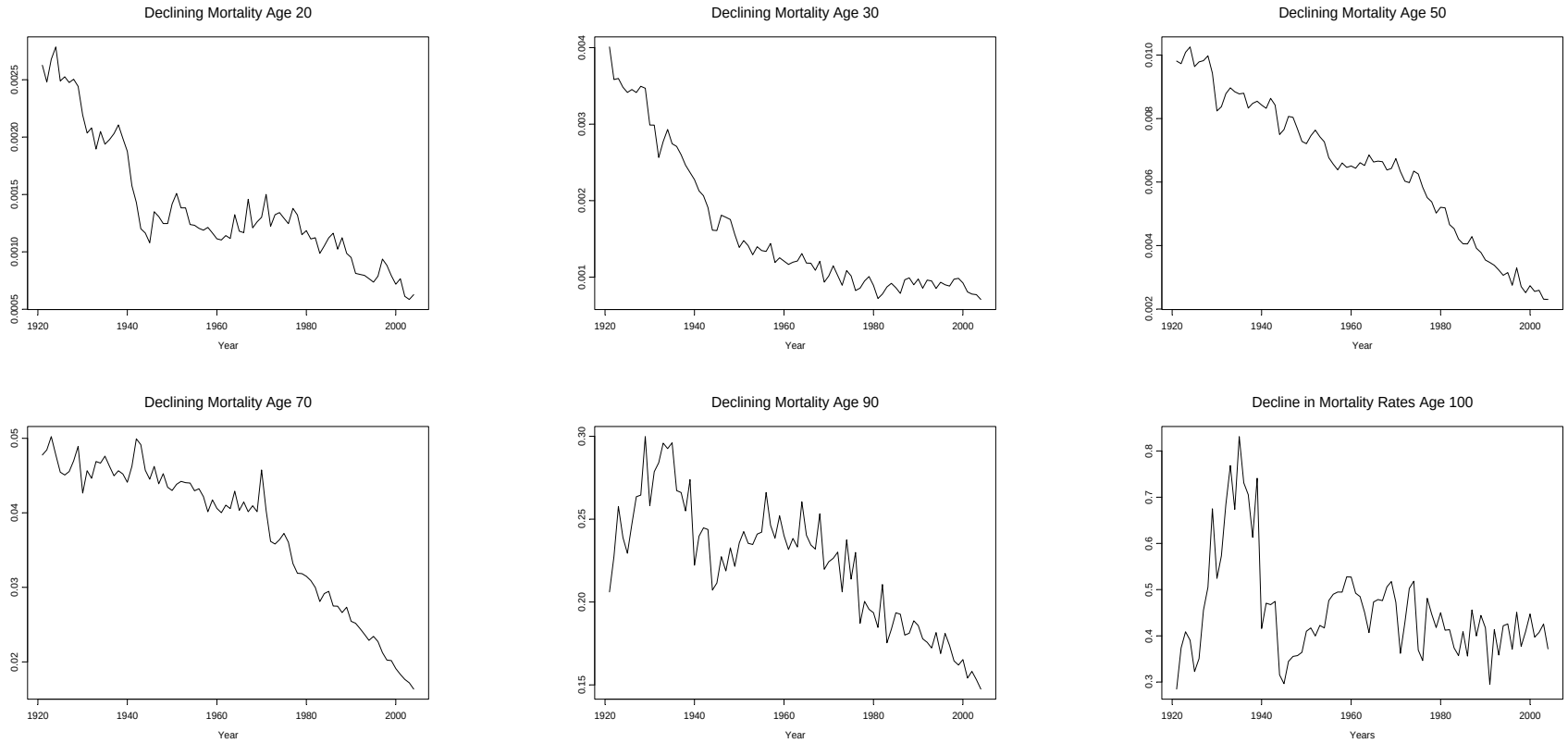


Figure: Age Specific Death Rates at Different Ages

Unit roots

Trend-stationary time series

$$y_t = \mu + \phi y_{t-1} + u_t \tag{1}$$

with $|\phi| < 1$

The random walk with drift

$$y_t = \mu + y_{t-1} + u_t \tag{2}$$

If $\phi = 1$ then the series has a unit root and as $T \to \infty$ the effect of the shocks persist and accumulate as stochastic trends in the series:

$$y_t = \mu + y_0 + \sum_{t=0}^{\infty} u_t \tag{3}$$

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Unit Root Tests

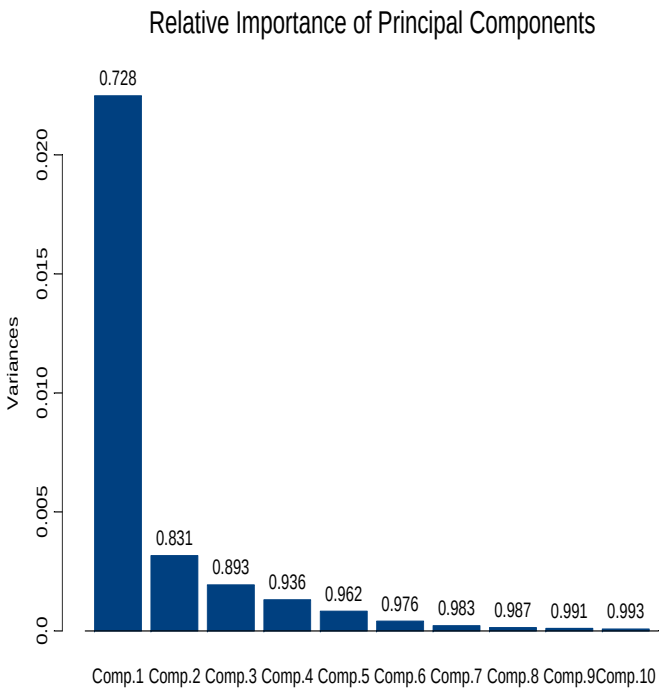
	Constant	Lags	Constant, Trend	Lags
Australia	0.9809	1	0.2383	1
Δ Australia	0	0	0	0
England	0.998	2	0.0345	0
Δ England	0	1	0	1
Japan	0.0129	1	$0.9374^{(1)}$	1
Δ Japan	0	0	0	0
Norway	0.9999	1	$0.9275^{(1)}$	1
Δ Norway	0	0	0	0
USA	0.8311	0	$0.8487^{(1)}$	0
Δ USA	0	0	0	0

Table: ADF Tests on Male Standardised Mortality Rates

$^{(1)}$ Indicates significant trend

Principal Components

Females



Males

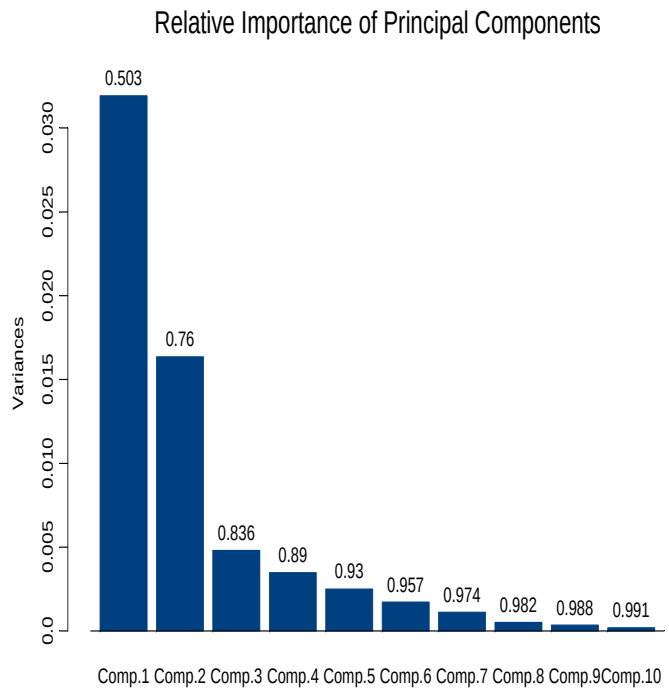
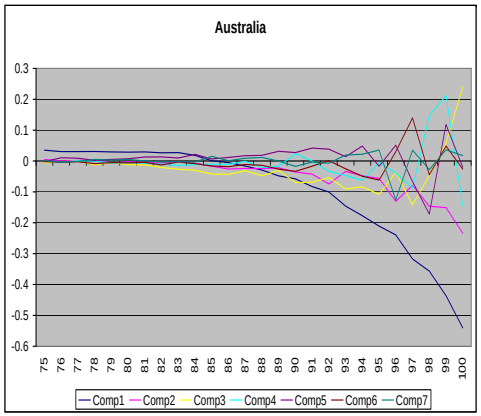


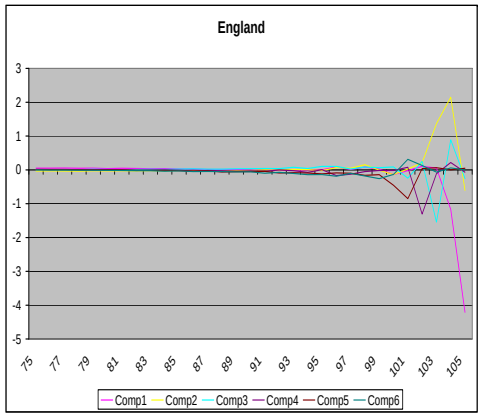
Figure: Principal Components for Australia - Mortality Rate Levels

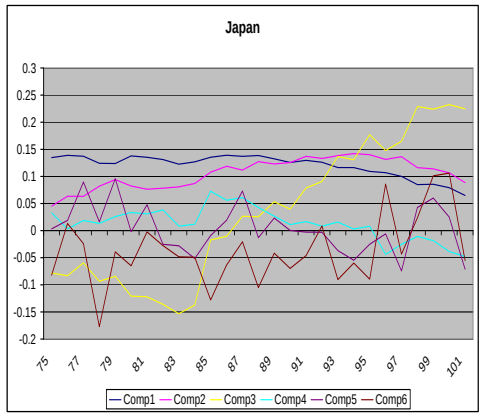
Country	Difference	Number of Factors	Percentage Variation
Australia	Diagonal	8	98.7
	Horizontal	8	98.6
England	Diagonal	7	98.6
	Horizontal	7	98.9
Japan	Diagonal	6	98.8
	Horizontal	5	98.1
Norway	Diagonal	9	98.4
	Horizontal	8	98.1
USA	Diagonal	10	97.4
	Horizontal	10	97.7

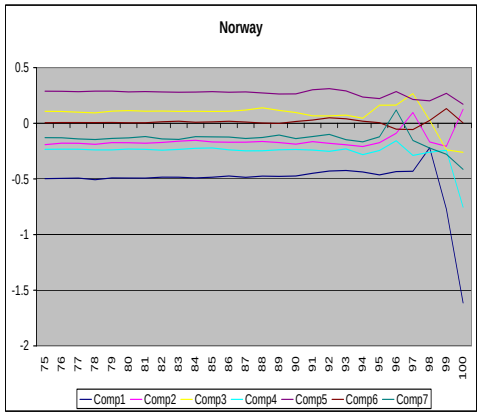
Table: PCA Factors using for differences in rates for countries in study

Principal Components









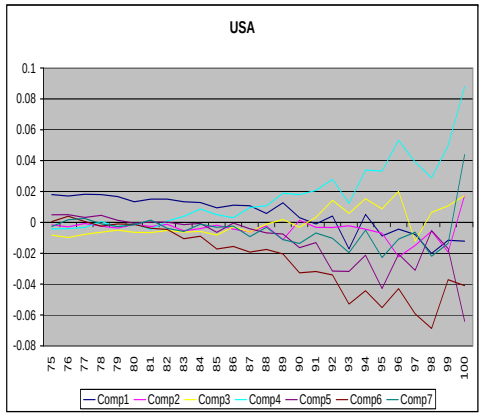


Figure: Multiple PCA Factors for Mortality Differences

VAR and VECM

VAR(p) for p lags:

$$m_t = A_0 + A_1 m_{t-1} + A_2 m_{t-2} + \cdots + A_p m_{t-p} + e_t$$

where $\mathbf{m}_t = (m_{1t}, \ldots, m_{kt}, \ldots, m_{Kt})$ for $k = 1, \ldots, K$ time series.
Long-run specification of VECM:

$$\Delta \mathbf{m}_t = \Gamma_1 \Delta \mathbf{m}_{t-1} + \cdots + \Gamma_{p-1} \Delta \mathbf{m}_{t-p+1} + \Pi \mathbf{m}_{t-p} + A_0 + e_t \quad (4)$$

where

$$\Gamma_i = -(I - A_1 - \cdots - A_i), \quad i = 1, \ldots, p-1 \qquad \Pi = -(I - A_1 - \cdots - A_p)$$

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VAR and Cointegration

VAR Models for standardised mortality rate across countries fitted

Autoregression in countries mortality rates.
Norway requires AR(2); other countries AR(1) and some cross country lags are significant

Countries in the study have stochastic trends with drift and no evidence of common stochastic trends (cointegration).

Heligman-Pollard (H-P) Model

$$q_x = A^{(x+B)^C} + D\exp[-E(\log\{\frac{x}{F}\})^2] + \frac{GH^x}{1+GH^x} \qquad (5)$$

	A	B	C	D	E	F	G	H
A	1.00	-0.03	0.65	0.07	0.24	-0.68	0.89	-0.85
B	-0.03	1.00	0.60	-0.17	-0.44	0.37	-0.29	0.32
C	0.65	0.60	1.00	0.09	0.05	-0.37	0.51	-0.47
D	0.07	-0.17	0.09	1.00	0.72	-0.55	0.25	-0.29
E	0.24	-0.44	0.05	0.72	1.00	-0.76	0.57	-0.62
F	-0.68	0.37	-0.37	-0.55	-0.76	1.00	-0.87	0.90
G	0.89	-0.29	0.51	0.25	0.57	-0.87	1.00	-0.98
H	-0.85	0.32	-0.47	-0.29	-0.62	0.90	-0.98	1.00

Table: Male Parameters Correlation Matrix

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H-P Model Parameters

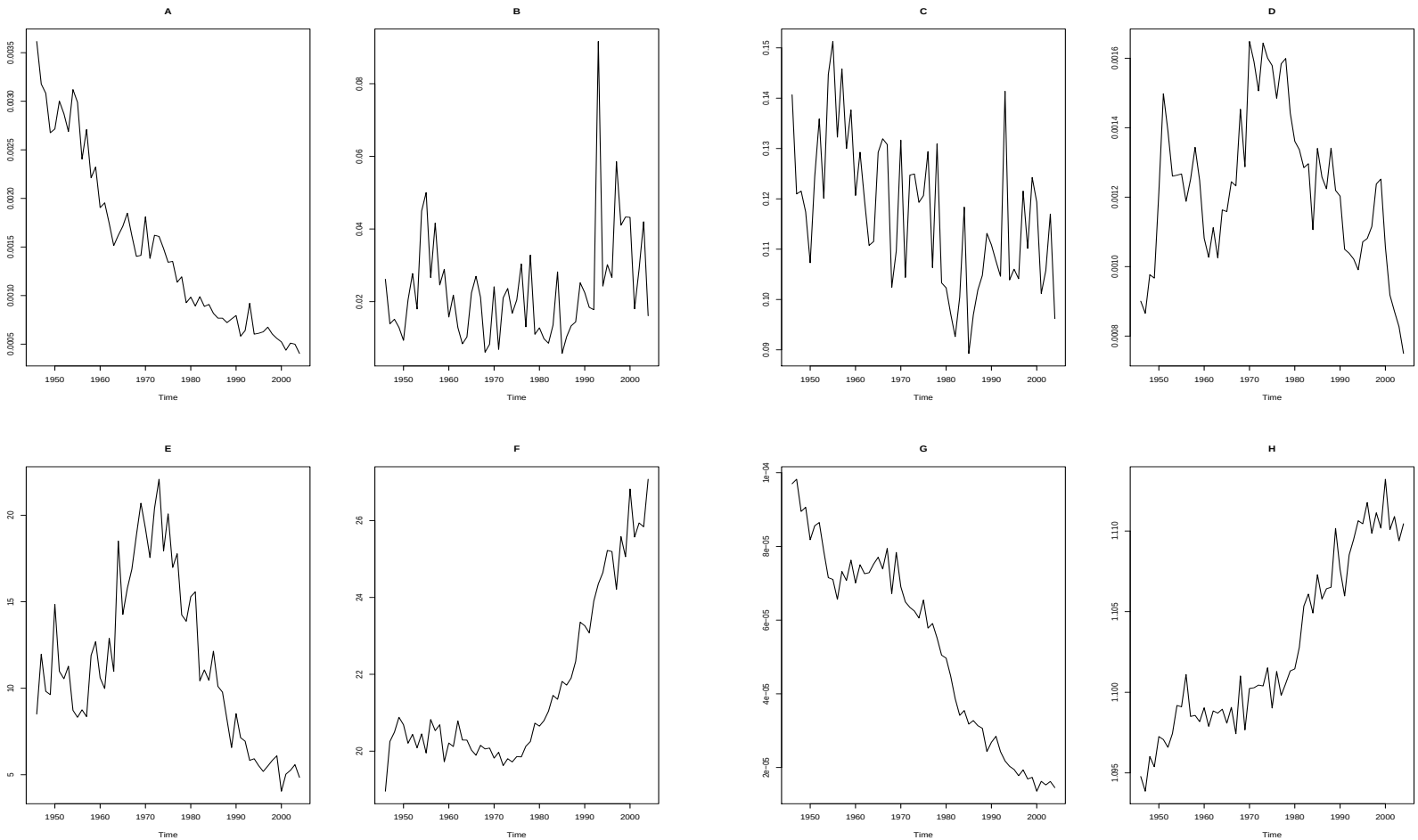

Figure: H-P Parameters for Australia - Males

Heligman-Pollard Model

There are common stochastic trends.

A VECM with one (at most two) cointegrating relationships captures the common stochastic trends.

Conclusions

Econometric analysis of standardized mortality rates for several major countries as well as for age specific rates shows:

Evidence of non-stationarity (stochastic trends and difference stationary) by country

Significant number of common factors in mortality rate levels as well as differences (multiple factor models for dimension reduction)

Mixture of non-stationary and stationary mortality rates by age and common stochastic trends across age groups in a country - Vector Error Correction models and Cointegration in a VAR model. Dimension reduction important in future modelling.