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Whither human survival and longevity
or
The shape of things to come

Longevity 5 NY September 2009

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Traditional method of projecting populations

- Project age-sex mortality rates
- Breakdown current population by age-sex
- The projected population for the following year is the current population plus assumed net immigration for the period plus births less deaths determined by mortality rates
- Continue projection for required number of years

Key demographic questions

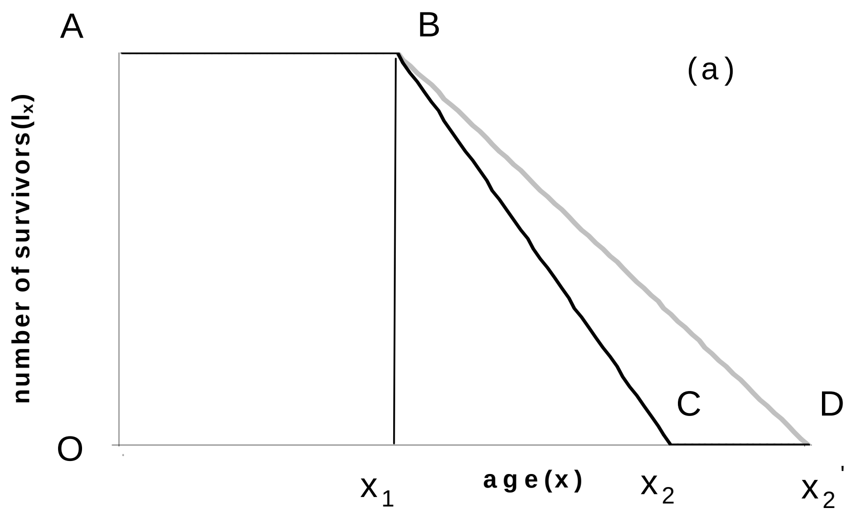
- How do populations age and grow?
- Is there any upper limit to age?
- Are there other methods we can use to project populations?
- Are these methods more accurate?
- What size of difference will it make?

http://www.cass.city.ac.uk/facact/research/reports/189ARP.pdf

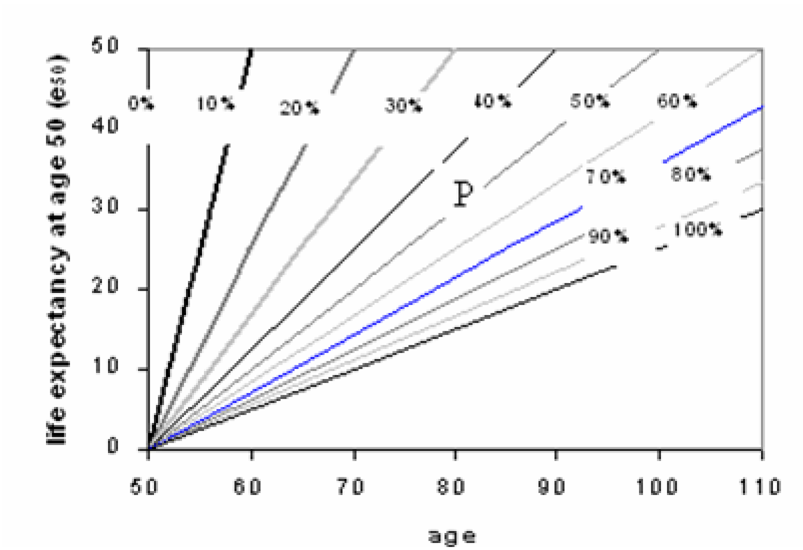
Aspects discussed

- Simple conceptual model which can be tested against data
- Empirical results from England and Wales
- Evidence that survival approach gives more accurate results than mortality based approach

Simple model



(i) The 'dispersed' survival model



(ii) Cumulative mortality as a function of life expectancy

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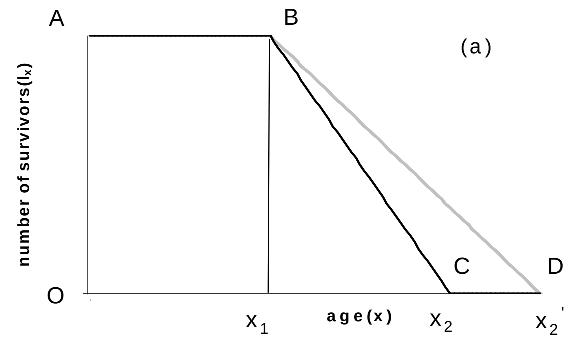
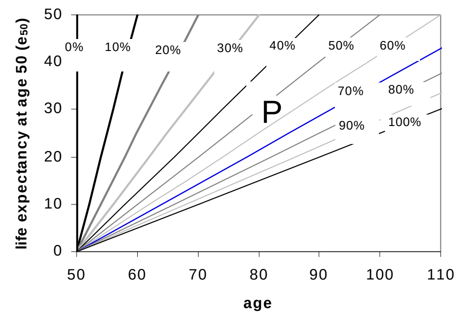
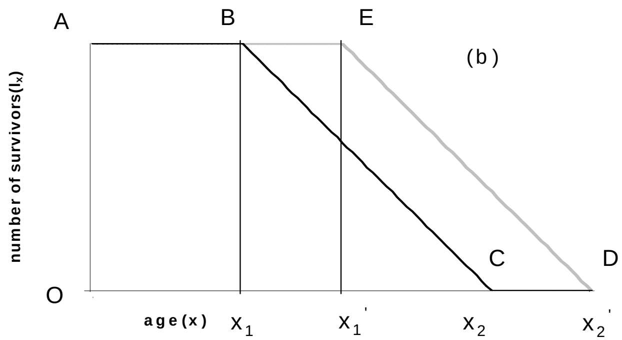
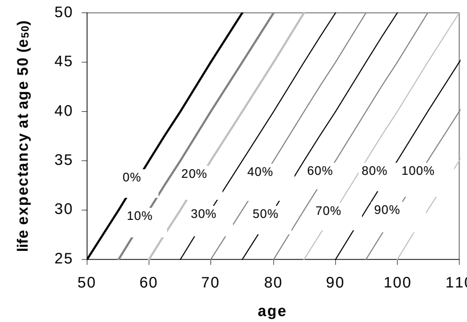
Model set



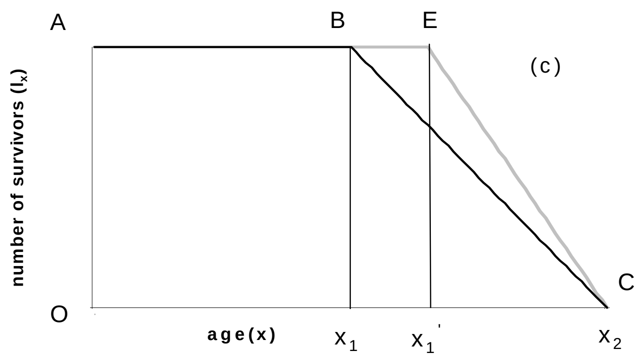


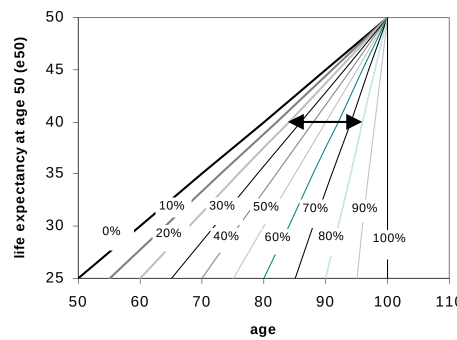
(a) Dispersed survival





(b) Parallel survival

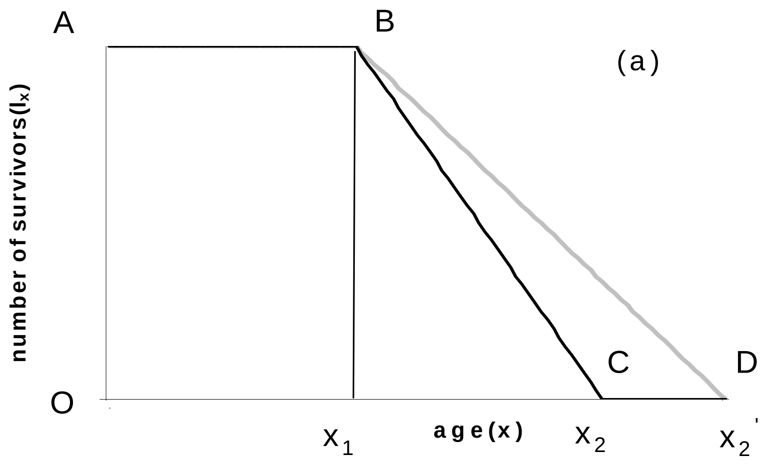


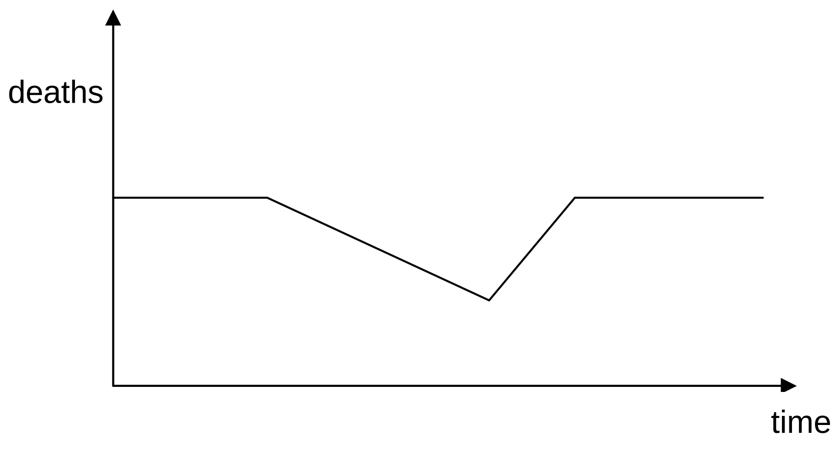


(c) Mortality compression

General observations

- Wide range of simply derived results
- leading to testable hypotheses
- and simple dynamics:





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Empirical results

- Applied to England and Wales from 1800 to 2003
- Looked for evidence of which model worked best in different time periods
- Looked for evidence of maximum age
- Used results to project UK population and compare accuracy

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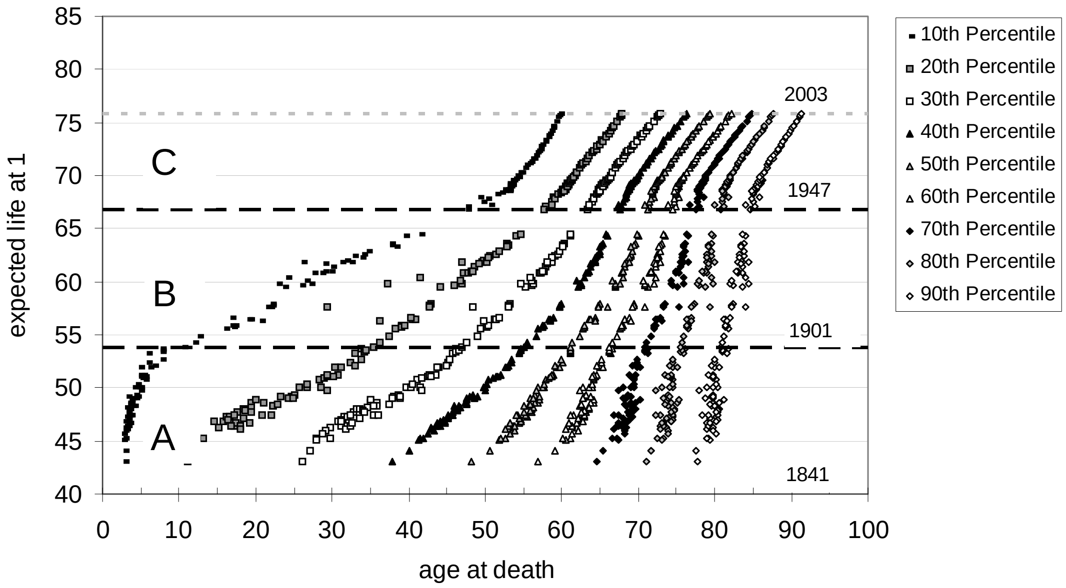
England and Wales life expectancy at age 1

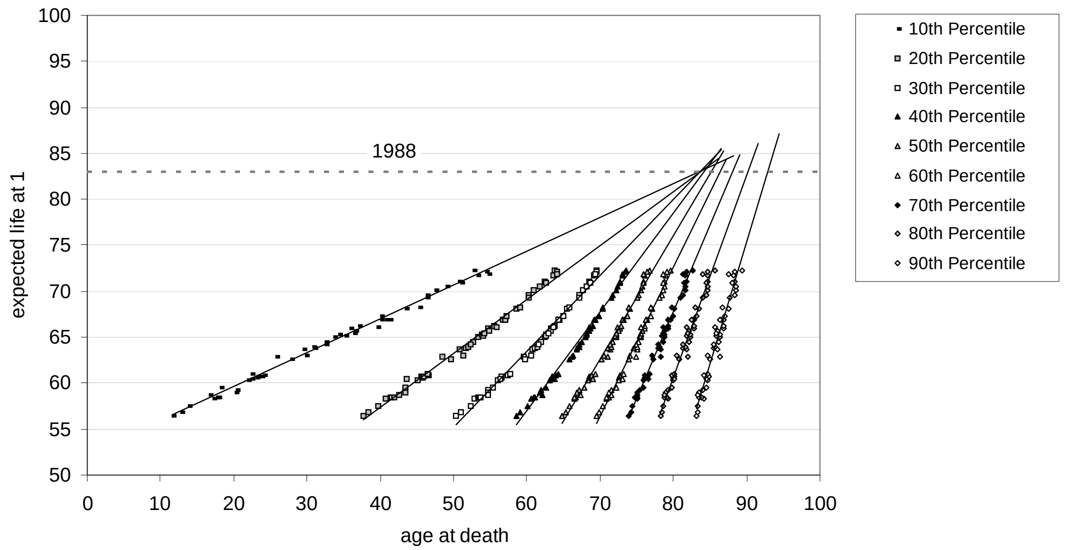


A – Starting in 1841 and ending in 1900 an era of rising life expectancy at age 1 from around 47 to 54 years (1.2 years per decade). Period of high infant and childhood mortality, but reducing health inequalities at older ages *(expansion hypothesis indicated)*

B - Starting in 1901 and ending in 1946, an era of rising life expectancy from around 54 to 67 years (2.8 years per decade). Period of falling infant and childhood mortality the continuing reduction in health inequalities *(compression hypothesis indicated)*

C - Starting in 1947 to the present, an era of continuing rises in life expectancy from 67 to 76 years (1.6 years per decade). Childhood mortality virtually eliminated so little improvement possible. Improvements in life expectancy due to more parallel behaviour so no longer convergence *(parallel hypothesis indicated)*

Towards a maximum age?



Convergent case based on females from England and Wales, 1901 to 1946 with fitted regression lines

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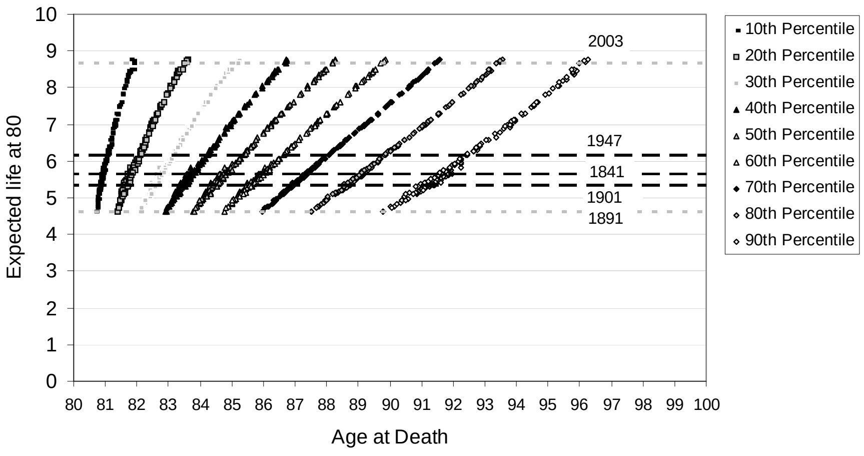
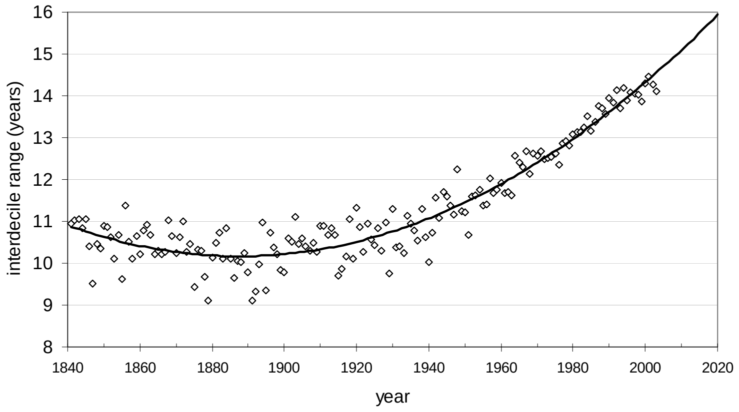
Convergence results

Converging percentiles	Age of death where convergence occurs	Required expected future lifetime at age 1	Calendar Year this occurs
10^{th} and 20^{th}	83.91	83.16	1988
20^{th} and 30^{th}	84.06	83.24	1989
30^{th} and 40^{th}	86.14	84.96	1994
40^{th} and 50^{th}	88.47	87.46	2003
50^{th} and 60^{th}	92.28	92.58	2019
60^{th} and 70^{th}	98.82	103.19	2055
70^{th} and 80^{th}	102.89	110.86	2080
80^{th} and 90^{th}	106.05	117.8	2103

But convergence pattern ceases after 1946 and so no maximum age is any longer indicated in the data so must look for reasons

Life expectancy at age 80





Inter decile range as a function of life expectancy at age 80 (females). *IDR is defined as the gap in years between 10th and 90th mortality percentiles*

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Population forecasting based on survival

__Procedure:__

1. Establish relationship between the percentiles and the expectation of life at a given age (we use age 50)
2. Establish nature of linear relationship between calendar year and expectation of life
3. Project forward expectation of life using the relationship found in stage 2
4. Derive survival percentiles using the projected expectation of life and the relationship derived in stage 1
5. Derive mortality from resultant life tables and proceed as normal

Gompertz Makeham equation

- A problem with using the full life table is the volume of data required.
- A more suitable solution can thus be to determine a function that fits the data so that only a few parameters are required.
- The function chosen to fit the data is a form of the Gompertz-Makeham Model which gives a very good fit to the data
- The Gompertz-Makeham Model provides a function for the force of mortality and is defined as:

$$\mu_x = A + Bc^x \text{ or } \mu_x = A + Be^{\gamma x} \text{ where } \gamma = \ln(c)$$

Where x = age and A, B and c are parameters

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Validity testing using retro-projections based on 1981

- To show suitability of the projections we compare projections from our model, with the GAD projection and actual survival from a 1981 base
- GAD's published projection was obtained by constructing life tables using the projected mortality rates at the time
- We use a survival approach but otherwise the same starting data as GAD's

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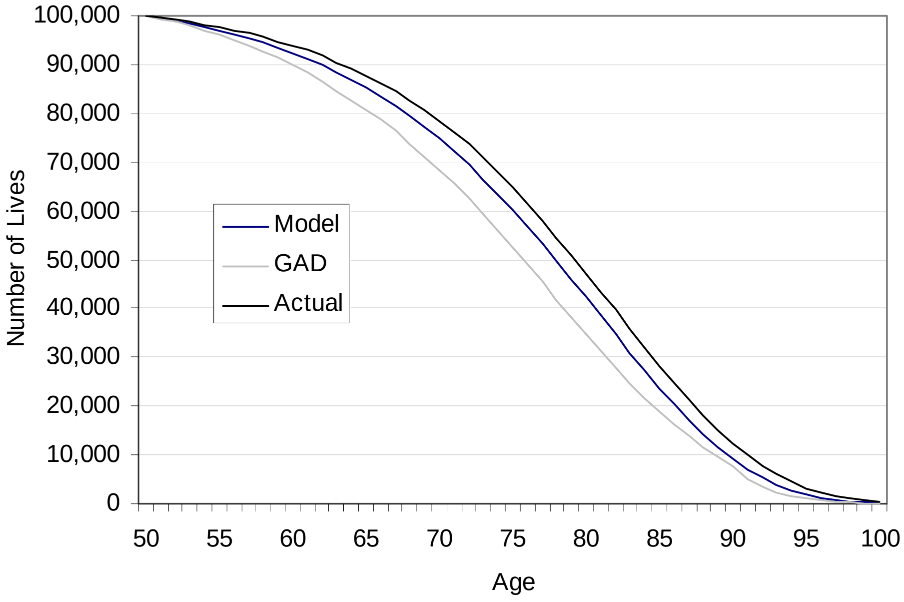
Predicted survival curves at age 50 for year 2000 using 1981 base



Comparison of projected life tables for 2000 for England and Wales males

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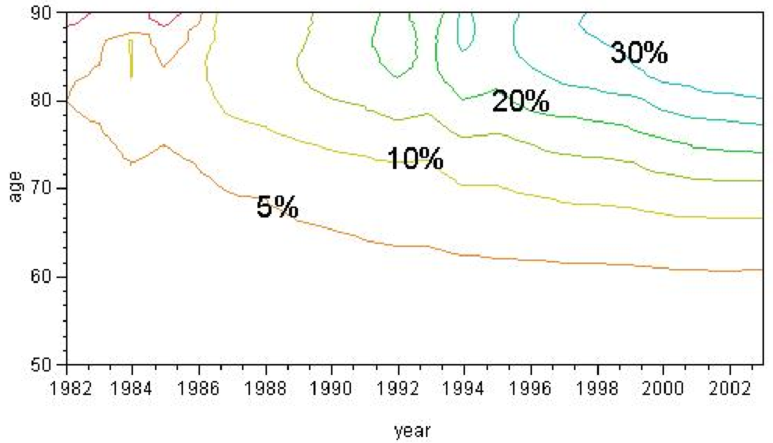
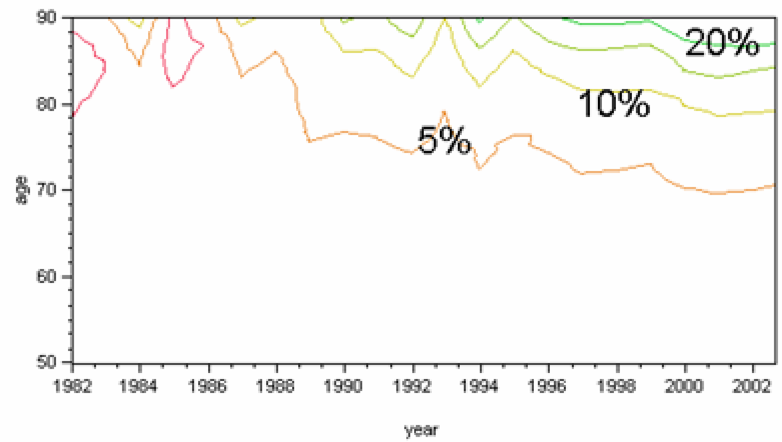
Comparison of percentage error between actual, GAD and model 1982 to 2003



(a) Percentage difference between Actual and GAD as a function of age and calendar year



(b) Percentage difference between Actual and Model as a function of age and calendar year

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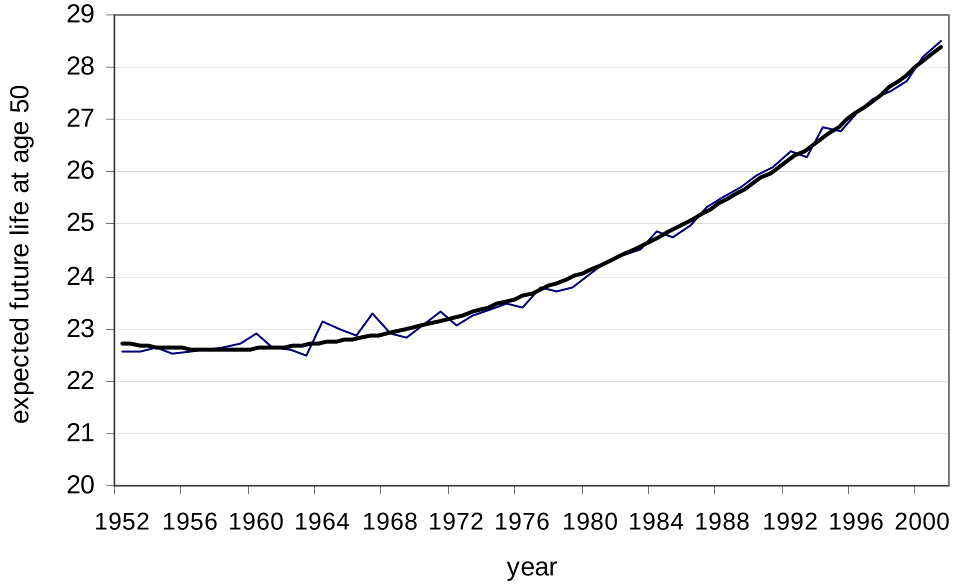
A comparison of 50+ population projections to 2020 based on GAD and on model ~ trend in male life expectancy at age 50



We used ARIMA techniques to check that projected life expectancy were within the 95% confidence interval

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A comparison of 50+ population projections to 2020 based on GAD and on model ~ base 2001

age	GAD 2020	Model	Diff	Diff %
50-59	3,788,205	3,809,512	21,306	0.56%
60-69	3,014,841	3,111,925	97,084	3.22%
70-79	2,324,314	2,504,966	180,653	7.77%
80-89	978,574	1,164,099	185,525	18.96%
total	10,105,934	10,590,502	484,568	4.79%

Difference for females was 110,858 on same basis, so total difference is nearly 0.6m people between GAD and model (equates to additional state pension costs of £3bn per annum)

Conclusions

- We have offered a new way of charting the development of populations through an analysis of trends in human survival based on ordinary life tables
- We sought to explain the various different shapes of survival curves starting with a simple model. This conjectured three basic patterns of survival: (a) ‘divergent’, (b) ‘convergent’ or (c) ‘parallel’
- There is currently no convergence towards a maximum age although this was not true before 1946. Extended survival patterns among the oldest old is one of the reasons
- Population projections based on survival trends rather than mortality trends produced more accurate projections of the England an Wales populations than GAD projections to 2003 from a 1981 base
- Population projections to 2020 using our model show 0.6m more people aged 50+ than GAD projections
- There is scope to develop the model in a range of applications and across countries