

Mortality-Indexed Annuities

Managing Longevity Risk via Product Design

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- Introduction: Motivation and Related Research
- Mortality-Indexed Annuities as a Product Design Proposal
- Simulation Framework and Benchmark
- Selected Results
- Conclusion



- **Demographic Transition** worldwide phenomenon (*Oeppen/Vaupel, 2002*)
 - Decreasing birth rates (*Berkel et al., 2002*)
 - Reason: Changing societal and family structures
 - Decreasing mortality (*Willets, 1999; Kytir, 2003*)
 - Reason: “better” living, working, environmental conditions; medical advances; health consciousness
- Consequences:
 - Changing age structures (age pyramids) (*Sinn, 2004*)
 - burden for PAYGO social security systems
 - Globally increasing life expectancies (*Vaupel, 1986; Oeppen/Vaupel, 2002*)
 - Societal achievement, also holds **longevity risk**



- **Individual Longevity Risk**

- Risk of individual deviations of lifetime from average. Sufficient financial means during retirement or post-working ages? (*MacMinn et al., 2006*)
- Social security tends to provide lower benefits than initially expected (*Schmähl, 2001*)
- Individuals challenged to adjust long-term saving/consumption to uncertain, longer lifetime (*Bloom et al., 2001*); possibly by transferring longevity risk to insurer (life annuity)

- **Aggregate Longevity Risk**

- Uncertainty regarding correct projection of future average mortality (*Blake/Burrows, 2001*)
- Strong, worldwide correlation (*Zahn/Henninger, 1942*); potential for accumulative losses
- Hardly diversifiable or (re)insurable (*Riemer-Hommel/Trauth, 2005*)



- **Annuity Puzzle:** empirically low demand for life annuities despite theoretical optimality (*Yaari, 1965*)
 - Several explanations exist in literature (*Davidoff et al., 2005; Brown/Orszag, 2006; Van de Wen/Weale, 2006; Schulze/Post, 2006; Milevsky/Young, 2007*)
 - Among others: prices could be too high or perceived to be excessive (*Mitchell et al., 1999; Murthi et al., 1999; Finkelstein/Poterba, 2002*)
 - partly justified due to strong correlation



- Conservative Pricing
 - Limited by competition, regulation
 - Limited marketability of excessively priced products (*e.g. Mitchell et al., 1999*)
(\Leftrightarrow tax advantages and other incentives designed to mitigate insufficient demand)
- Natural Hedging (*Cox/Lin, 2007; Wetzel/Zwiesler, 2008*)
- Securitization
- Leaving the annuity market (?)
- **Modification of actuarial product design**



- **Example:** private health insurance in Germany
 - Design similar to life annuities: recurring, constant premiums; lifelong coverage
 - Policyholders bear systematic risk of increasing health expenditures (**premium adjustments**)
- Also: Transfer of risk successful with respect to investment risk (e.g., unit-linked life insurance/life annuities)
- **Proposal: Mortality-Indexed Annuity (MIA)**
as modification of a constant life annuity
 - **New:** adjustments of annuity payments based on **actual** mortality experience: higher/lower portfolio mortality → higher/lower benefits
- **Result:** limited risk for insurer; policyholders' perspective?



- Immediate annuity sold against (actuarially fair) single premium; constitutes initial per-policy reserve
- Evolution of reserves due to **inheritance effect** and **interest**
- **Annual** adjustments of benefits according to equivalence principle (best estimate of mortality, based on actual **portfolio** experience)
- **Regulatory requirements** neglected (taxation, calculation requirements, model choices etc.)
- Further details
 - No period certain
 - Constant interest rate
 - Pure net perspective without costs or expenses; actuarially fair price



- **Monte-Carlo simulation** ($N=10,000$ paths)
 - Consider a large portfolio of homogeneous risks over T periods
- **General** mortality follows **Lee-Carter model**
(*Lee/Carter, 1992; Brouhns et al., 2002*)
- Best estimate of mortality for remaining periods based on Lee-Carter, accounting for mortality **experience**.
- Mortality data of British annuitants, Source: CMI
- Males, initial age $x=60$, single premium $\pi_0=100,000$, contract term $T=41$ (last payment due on 100th birthday)



- Idea: constant life annuity with guaranteed benefits serves as a benchmark – identical single premium π_0
- Starting point: Initially ($t=0$), benchmark benefits equal to those from MIA, as calculation based on identical assumptions; but: benefits reduced by safety loading (see below)
- Mortality correctly projected **on average**, but subject to **uncertainty**
 - Calculation sufficient **on average**, but underlies **strong fluctuations**
 - Insurer charges **safety loading** to reduce deficit risk to α



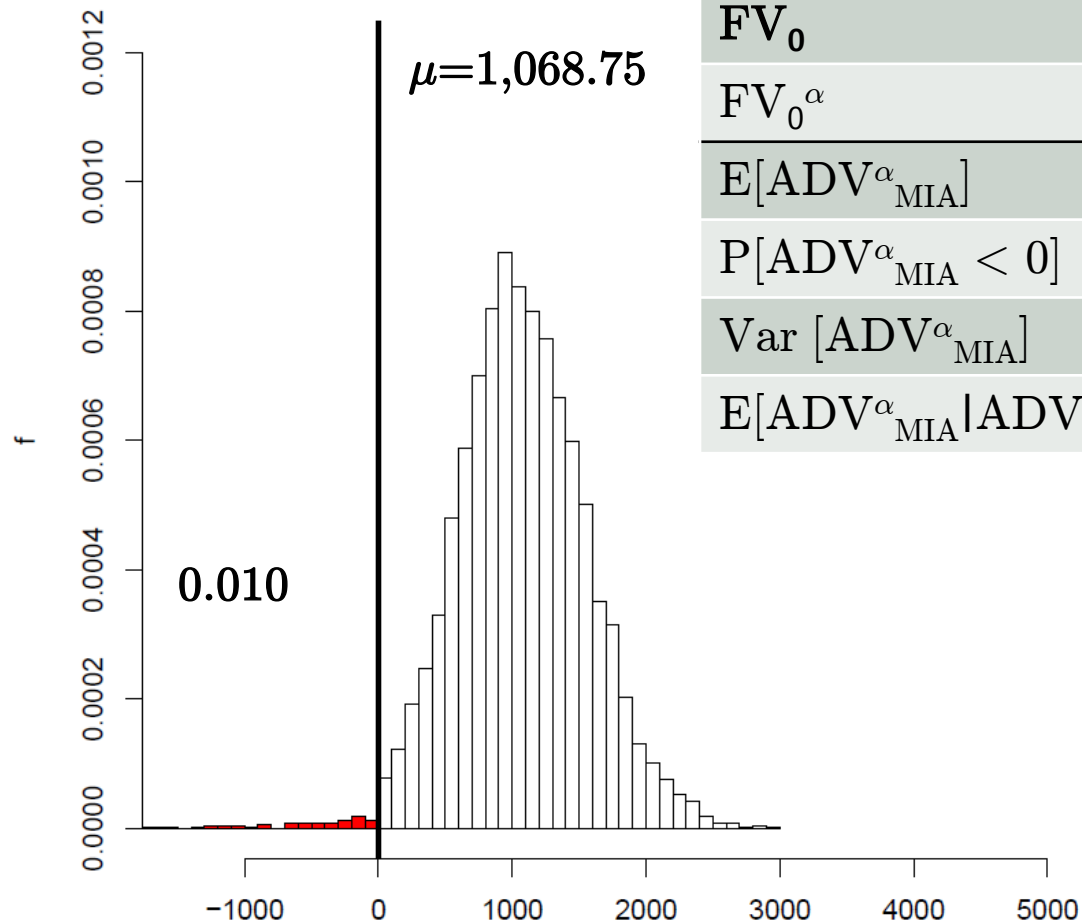
- Single premium assumed fixed (identical price π_0), **benefits reduced** from FV_0 to FV_0^α to incorporate safety loading
 - Difference accumulated over contract term in order to reduce deficit risk
- Large potential for surplus reserves; increased by safety loading
 - **Pro-rata** surplus share for policyholders
e.g. **X=75%** (in Germany since 2008)



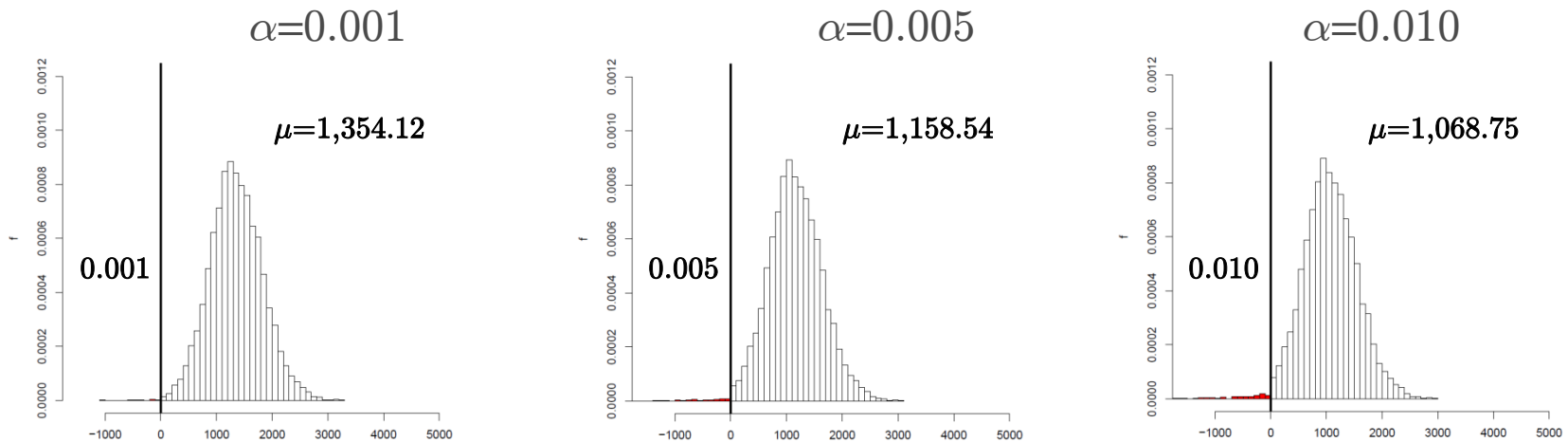
- Measure of “advantageousness”: **actuarial present value** of differences of benefits from both products, subject to **actual** mortality:

$$ADV_{MIA}^{\alpha} = \sum_{k=0}^{T-1} \{(FV_k - FV_0^{\alpha}) \cdot {}_k\tilde{p}_x \cdot v^k\} - X \cdot \max\{0; V_{T,\alpha}^{*,\text{conv}}\} \cdot {}_{T-1}\tilde{p}_x \cdot v^{T-1}$$

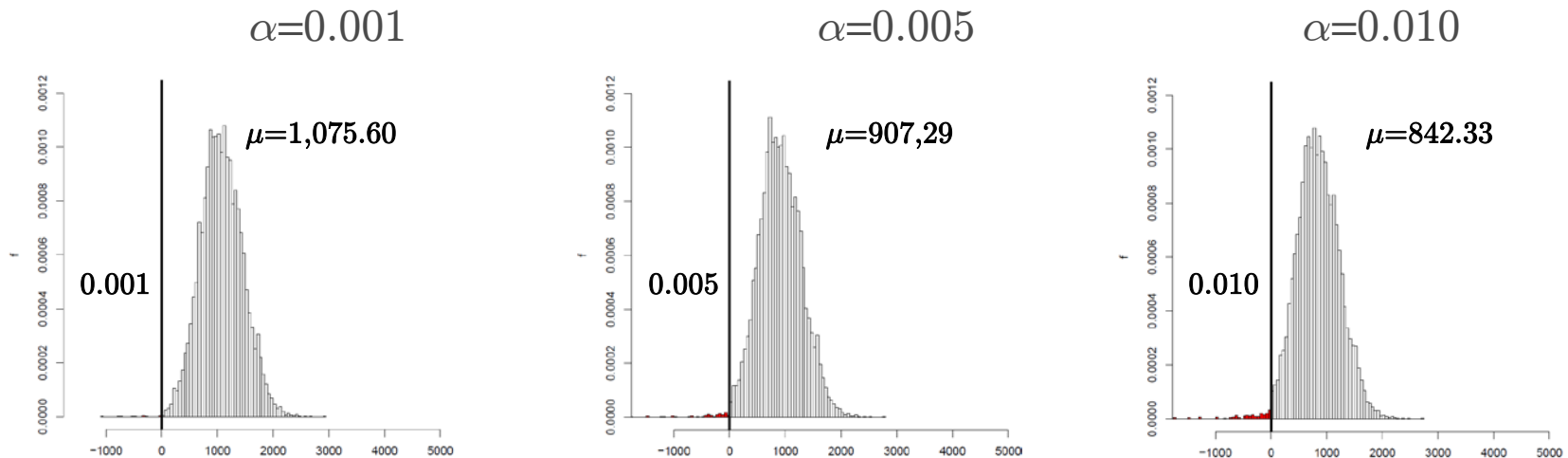
- Consider **empirical distribution/coefficients** of ADV_{MIA}^{α}



| | |
|--|-----------|
| FV_0 | 5,631.67 |
| FV_0^{α} | 5,393.50 |
| $E[ADV_{MIA}^{\alpha}]$ | 1,068.75 |
| $P[ADV_{MIA}^{\alpha} < 0]$ | 0.010 |
| $Var [ADV_{MIA}^{\alpha}]$ | 238,233.3 |
| $E[ADV_{MIA}^{\alpha} ADV_{MIA}^{\alpha} < 0]$ | -520.37 |



| | $\alpha = 0.001$ | $\alpha = 0.005$ | $\alpha = 0.010$ |
|--|------------------|------------------|------------------|
| FV_0 | 5,631.67 | | |
| FV_0^α | 5,330.00 | 5,373.50 | 5,393.50 |
| $E[ADV_{MIA}^\alpha]$ | 1,354.12 | 1,159.54 | 1,068.75 |
| $P[ADV_{MIA}^\alpha < 0]$ | 0.001 | 0.005 | 0.010 |
| $Var [ADV_{MIA}^\alpha]$ | 220,915.5 | 229,915.0 | 238,233.3 |
| $E[ADV_{MIA}^\alpha ADV_{MIA}^\alpha < 0]$ | -321.29 | -492.31 | -520.37 |



| | $\alpha = 0.001$ | $\alpha = 0.005$ | $\alpha = 0.010$ |
|--|------------------|------------------|------------------|
| FV_0 | 6,975.56 | | |
| FV_0^α | 6,679.05 | 6,725.70 | 6,743.45 |
| $E[ADV_{MIA}^\alpha]$ | 1,075.60 | 907.29 | 842.33 |
| $P[ADV_{MIA}^\alpha < 0]$ | 0.001 | 0.005 | 0.010 |
| $Var [ADV_{MIA}^\alpha]$ | 142,381.3 | 147,804.8 | 152,138.2 |
| $E[ADV_{MIA}^\alpha ADV_{MIA}^\alpha < 0]$ | -448.61 | -413.32 | -391.25 |

 $i=3\%$, $l_{60}=100,000$

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|--|------------------|------------------|------------------|
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 $i=3\%$, $l_{60}=100,000$ $i=3\%$, $l_{60}=1,000$

| | | | |
|--|-----------|-----------|-----------|
| FV_0 | 5,631.67 | | |
| FV_0^α | 5,285.50 | 5,334.00 | 5,358.50 |
| $E[ADV_{MIA}^\alpha]$ | 1,552.15 | 1,335.36 | 1,224.22 |
| $P[ADV_{MIA}^\alpha < 0]$ | 0.001 | 0.005 | 0.010 |
| $Var [ADV_{MIA}^\alpha]$ | 274,235.1 | 285,507.6 | 297,112.2 |
| $E[ADV_{MIA}^\alpha ADV_{MIA}^\alpha < 0]$ | -329.35 | -501.95 | -577.94 |

 $i=3\%$, $l_{60}=1,000$



- Longevity risk creates highly correlated long-term contractual obligations for insurance companies.
- If longevity risk is considered a severe threat to insurability, alternative product design and risk (re)transfer to policyholders should be considered.
- MIA transfer a significant amount of risk to policyholders, but in return ensure insurability and offer substantial upside potential.
 - Mostly greater annuity payments, expected advantages strictly positive.
 - The more expensive the benchmark, the more advantageous the MIA.
 - The lower the interest rate, the stronger the MIA advantage.
 - A smaller insured portfolio increases the safety loading required by the benchmark product.



- Refined actuarial modeling:
 - **Stochastic investment returns** from diversified portfolio
 - **Model uncertainty**: insurer does not know “true nature”
 - Benefit only adjustments beyond certain **thresholds**
 - **Adjustments** to mortality **index** (→ transparency vs. basis risk)
- Policyholders’ **risk aversion**: (transferred) risk vs. (higher) benefits
→ more accurate analysis of **risk allocation** effects
- More explicit modeling of defaults

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