Longevity risk and hedge effects in portfolios of life insurance products with investment risk

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Introduction

Measuring longevity risk

- Future survival probabilities are stochastic $\longrightarrow$ uncertainty in payments of life insurance products;
- Longevity risk is often quantified by distributional characteristics of the discounted cash flows, assuming a constant and deterministic interest rate, r.
- Problem: interpretation?
  - In real world there is investment risk;
  - Level of future payments is uncertain $\longrightarrow$ cannot (fully) hedge against investment risk;
  - When return is as least r, this risk measure is a conservative one.
- We use an asset-liability approach to quantify longevity risk.

Longevity risk

- Focus on the effect of systematic longevity risk in a portfolio of life insurance products:
  - gender composition;
  - survivor annuities;
  - death benefits.
- Effect of investment risk when there exists longevity risk:
  - longevity risk $\leftrightarrow$ investment risk;
  - investment risk $\rightarrow$ hedge potential.
- Hedge effects of survival swaps:
  - survivor annuities;
  - basis risk.

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Model

Evolution of assets

- Risk measure based on: $\mathbb{P}\left(A_T < 0\right)$.
- Evolution of the assets:
  R_{s+1} return, $\widetilde{L}_s$ liability payment

$$A_{s+1} = \left(A_s - \widetilde{L}_s\right) \cdot \left(1 + R_{s+1}\right), \quad \text{for } s = 0, \cdots, T.$$

- Terminal asset value:

$$A_T = A_0 \cdot \prod_{s=1}^{T} \left(1 + R_s\right) - \sum_{s=1}^{T} \widetilde{L}_s \cdot \prod_{\tau=s+1}^{T} \left(1 + R_\tau\right).$$

- Capital requirement: required initial asset value in excess of the best estimate

$$A_0 = (1 + c) \cdot BEL.$$

Risk measure

- Investment portfolio is decomposed in:
  - Best estimate portfolio, with return r_s^{be}; Invested in zero-coupon bond $\longrightarrow$ eliminates interest rate risk in best estimate scenario;
  - Buffer portfolio, with return r_s^{bu}.
- Probability of ruin:

$$\mathbb{P}\left(A_T < 0\right) = \mathbb{P}\left(L > (1+c) \cdot BEL\right),$$

$$L \equiv BEL + \sum_{s=1}^{T} \left(\frac{\widetilde{L}_s - \mathbb{E}\left[\widetilde{L}_s\right]}{\prod_{\tau=1}^{s} \left(1 + r_\tau^{bu}\right)} \right).$$

Note: interaction longevity and investment risk!

- Capital requirement in excess of best estimate:

$$c = \frac{\mathbb{Q}_{1-\epsilon}\left(L\right)}{BEL} - 1.$$

Sources of risk

Focus on non-hedgeable risks: Best estimate portfolio invested in bonds.
We decompose L into four components:

i) Best estimate of the liabilities;
Deterministic, no uncertainty.

ii) Pure longevity risk component;
Uncertainty in L given expected returns.

iii) Pure investment risk component;
Uncertainty in L given expected cash flows.
Hedgeable risk: we generally set this equal to zero.

iv) Interaction investment and longevity risk component;
Uncertainty in L which is not captured by the other components.
Additional uncertainty!

- Life insurance products:
  - i) Single life annuity;
  - ii) Survivor annuity;
  - iii) Death benefit.
- Uncertainty in forecasting models includes: Process risk, parameter risk, and model risk.
- Longevity risk:
  - Variants of Lee-Carter (1992)-model;
  - Variants of Cairns-Blake-Dowd (2006)-model;
  - P-Splines model (Currie, Durbin, and Eilers; 2004).
- Investment risk:
  - Bond prices: Vasicek model;
  - Stock prices: Brownian motion with drift.

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Effect portfolio mix

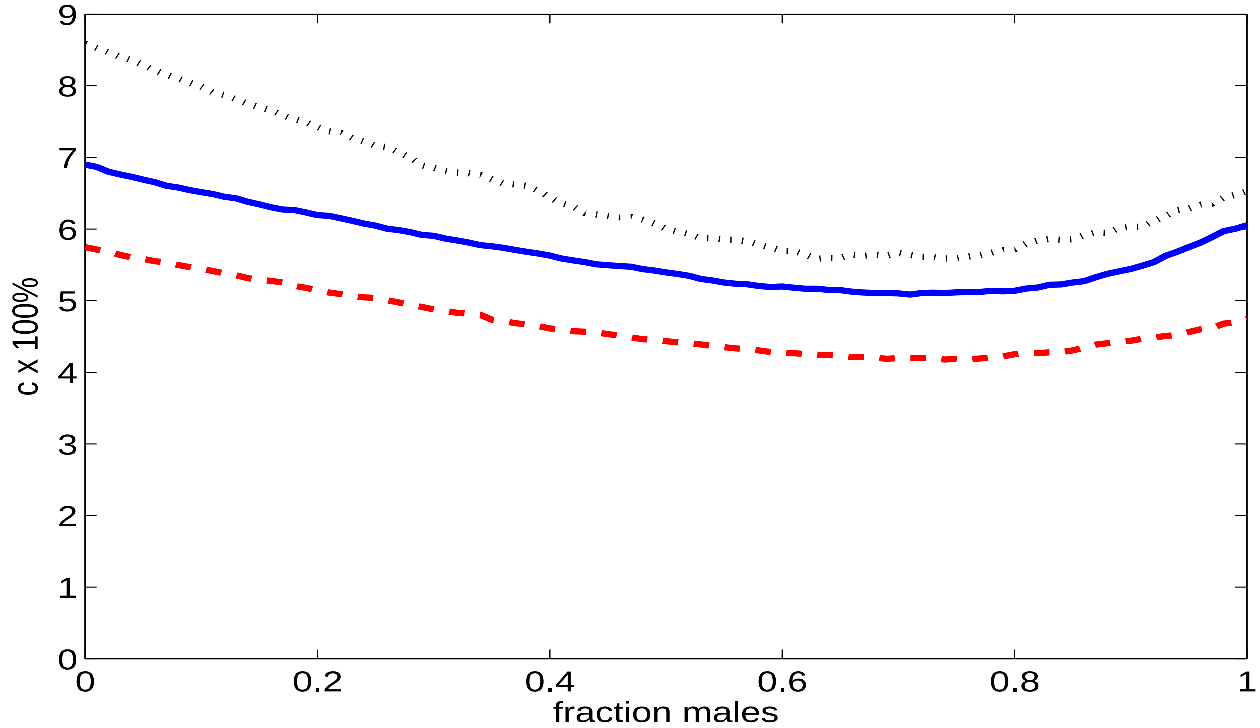
Effect interest rate risk

- Buffer requirement depends on investment strategy and liability portfolio of insurer.
- Buffer portfolio invested in one-year bonds, best estimate portfolio:
  - Only one-year zero-coupon bonds;
  - Portfolio of bonds eliminating pure investment risk.
- Buffer requirements:

Product	$c^{1y,1y}$	$c^{elh,1y}$
Male single life annuity	27.6%	6.1%
Female single life annuity	33.0%	6.9%

- We observe:
  - Investment risk might be large.

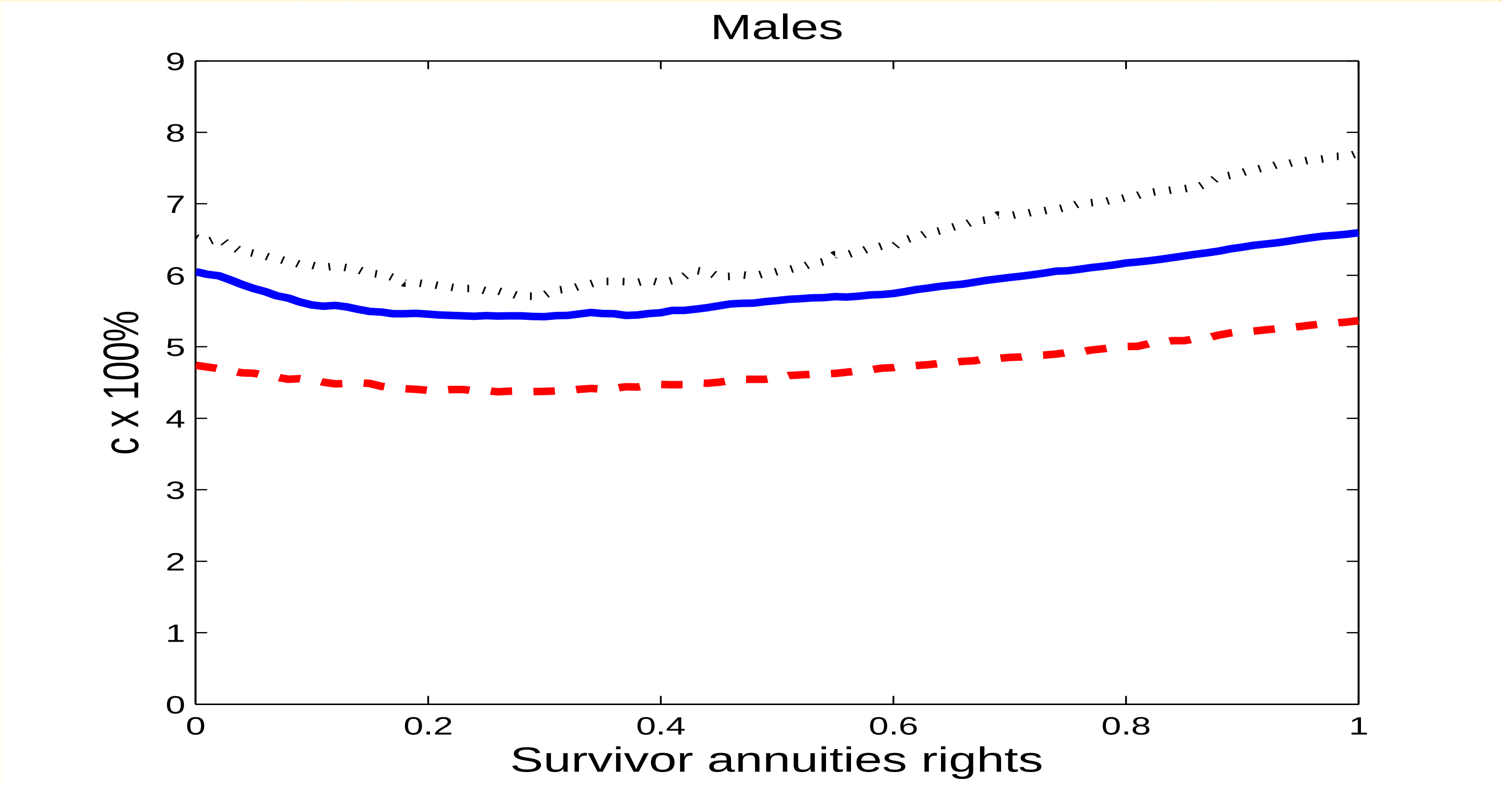
Effect gender mix



Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- Longevity risk is higher for females than males;
- Investment risk significantly affects required buffer;
- Higher duration typically increases impact of investment risk.

Effect product mix



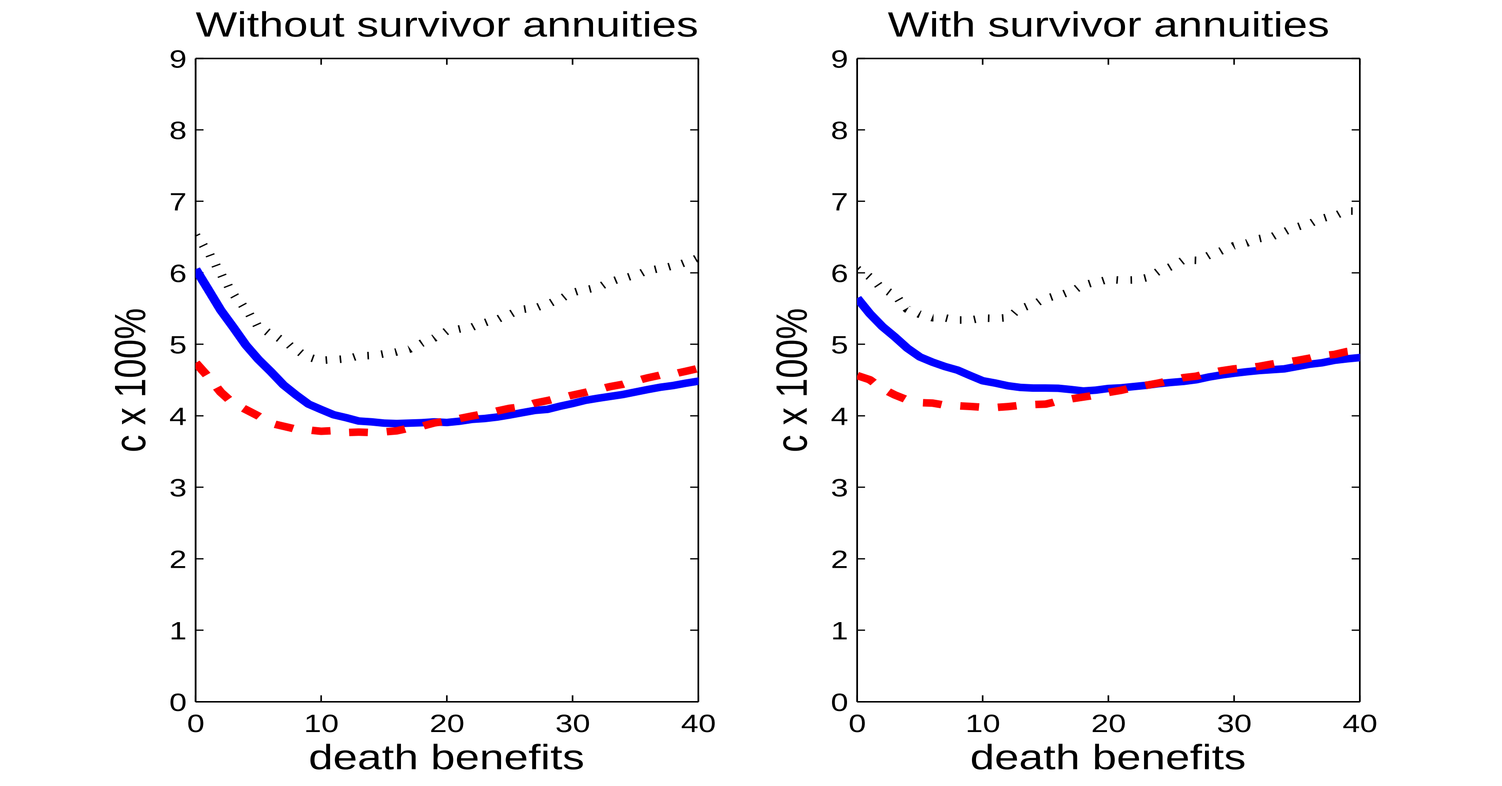
Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- Survivor annuities can reduce longevity risk;
- Effect of hedge potential depends significantly on investment risk.

Effect death benefits I

- Investment risk affects hedge potential.
- Death benefit consists of a single payment at the moment the insured dies.
  $\longrightarrow$ effect of living longer = postponing payment.
- Hedge effects of death benefits are due to discounting effects!
- When there is no investment risk:
  The aggregate payments of the portfolio is independent of the number of survivors when:
  - single life annuity with a yearly payment of 1;
  - death benefit with a single payment $\delta = \frac{1+r}{r}$.

Effect death benefits II



Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

- As expected: death benefits can reduce longevity risk;
- Survivor annuities reduce hedge effects of death benefits;
- Hedge effects of death benefits are significantly influenced by investment risk.

Effect survivor swaps

- Mortality linked assets can reduce longevity risk.
- An often proposed product is a survivor swap or longevity bond.
- The payments of a survivor swap in year s are given by:

$$SS(s, ref) = S(s, ref) - K(s, ref).$$

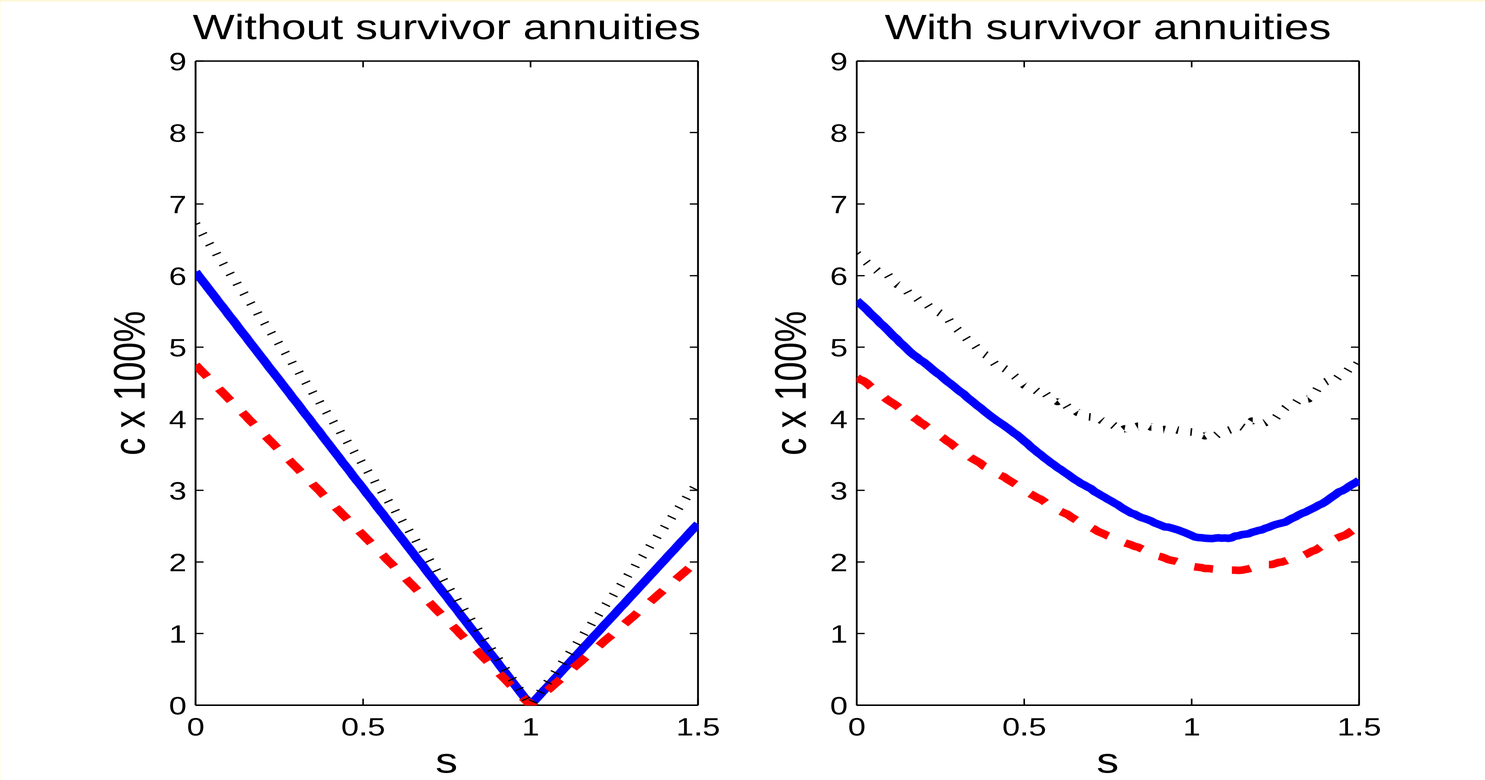
- Without basis risk: survivor swaps can provide a perfect hedge for single life annuities;
- How well do they work:
  - In a portfolio of life insurance products?
  - When there is basis risk?
  - When there is investment risk?

- Survivor swaps will have a (not observable) price;
- Attractiveness of swaps depend on:
  - Hedge effect of survivor swap;
  - Price of the survivor swap.
- We do not observe a market price $\longrightarrow$ we set reserve requirements in excess of best estimate of the liabilities & price of the swap.
  Let $V_{VSS}\left(s_m, s_f\right)$ be the price of the swap:

$$A_0 = BEL + V_{VSS}\left(s_m, s_f\right) + \overline{c}\left(s_m, s_f\right) \cdot BEL.$$

- Risk reduction can be interpreted as a maximum price.

No basis risk



Solid curve: 100% bonds; dashed curve: 1/3 equity, 2/3 bonds; dotted curve: 100% equity.

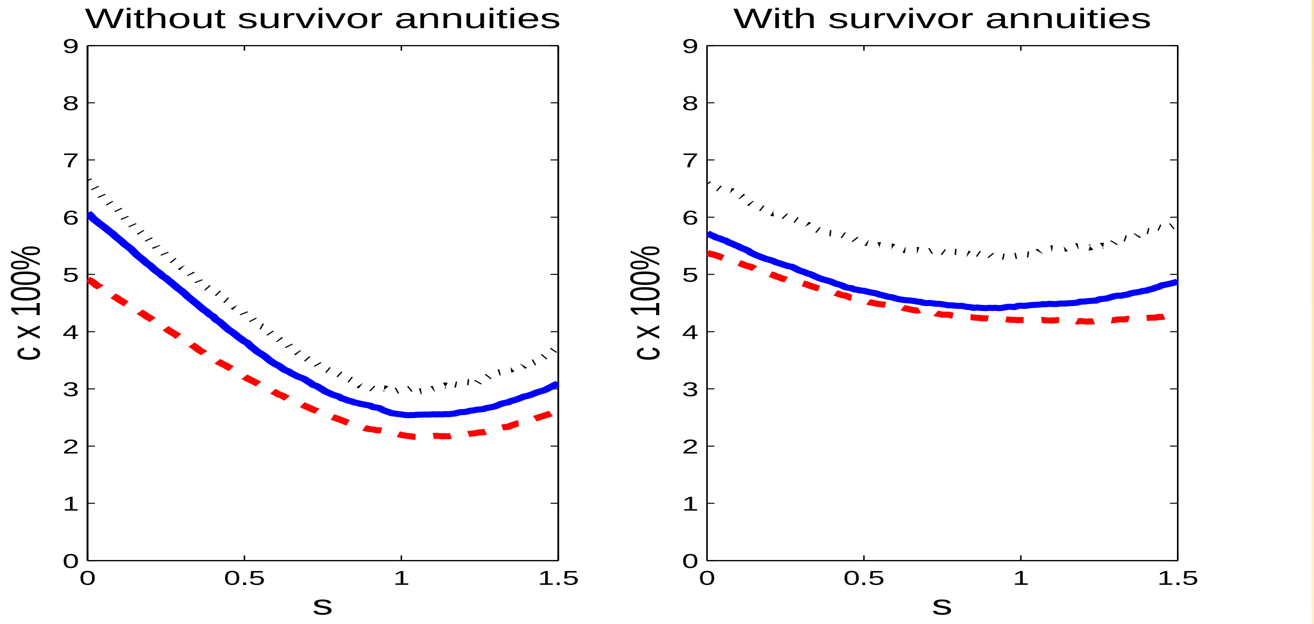
- Survivor annuities can significantly affect hedge potential of survival swaps;
- Maximum number of swaps depends on investment risk;
- With survivor annuities: investment risk generally reduces hedge effect of swaps.

Basis risk

- Mortality rates of insureds and general population differ.
- Denuit (2008) uses a Cox-type relational model:

$$\log(\mu_{x,t}^{(h)}) = \alpha^{(h)} + \beta^{(h)} \cdot \log(\mu_{x,t}^{(g)}),$$

with $\alpha^{(m)} = -1.54, \alpha^{(f)} = -1.02, \beta^{(m)} = 0.82, \beta^{(f)} = 0.91.$



- Basis risk significantly affects hedge potential of survival swaps.

Conclusions

Conclusions

- Liability only approach might significantly underestimate reserve requirements;
- Product and gender mix affects required buffer;
- Investment risk affects hedge potential:
  - Gender mix and survivor annuities: A higher duration typically leads to a stronger effect of investment risk.
  - Death benefits: More investment risk leads to a weaker hedge potential of death benefits.
  - Survivor swaps: Depends on the liability portfolio.
- Basis risk might significantly reduce the hedge potential of survivor swaps.