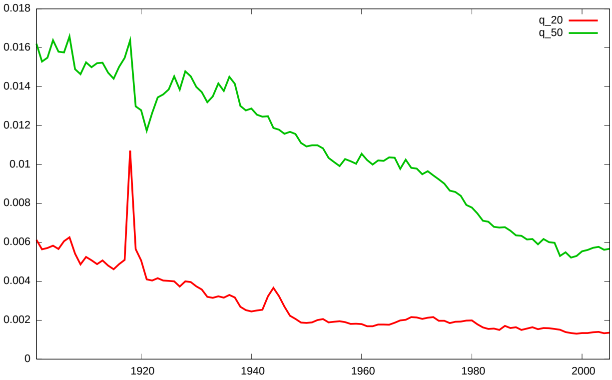

Risk and Valuation of Mortality Contingent Catastrophe Bonds

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Related Literature

Market and Securities

Model and Estimation

Results

Conclusion

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- Cox, Lin & Wang (2006,JRI), Lin & Cox (2008,IME)
  - → pricing models for mortality-contingent securities in incomplete market framework
  - → model for mortality index: GBM & multiplicative jump component
- Chen & Cox (2009,JRI)
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  - → Lee-Carter extensions with multiplicative jump component
- Cox, Lin & Milidonis (2009)
  - → pricing models for mortality-contingent securities in incomplete market framework
  - → regime-shifting models

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- Cox, Lin & Milidonis (2009)
  - → pricing models for mortality-contingent securities in incomplete market framework
  - → regime-shifting models
- Here:
  - risk assessment, comparison to official loss profiles
  - "endogenous" valuation (similar to Lin & Cox (2005,JRI); Bauer, Börger & Ruß (2009,IME)) → problems...
  - → affine mortality model with additive jump components

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Issued	Vita Capital Ltd. Nov. 2003	Vita Capital II Ltd. Apr. 2006			Tartan Capital Ltd. May 2006	
Class[5]	A	B	C	D	A*	B
Tranche Size	$400mn	$62mn	$200mn	$100mn	$75mn	$80mn
Arranger	Swiss Re	Swiss Re			Goldman Sachs	
Protection for	Swiss Re	Swiss Re			Scottish Re	
Rating[6]	A3/A+	Aa3/A-**	A2/BBB+**	Baa2/BBB-**	Aaa/AAA	Baa3/BBB+
Attachment Point	130%	120%	115%	110%	115%	110%
Detachment Point	150%	125%	120%	115%	120%	115%
Coupon (bps)	LIBOR+135	LIBOR+90	LIBOR+140	LIBOR+190	LIBOR+19	LIBOR+300
Expected Maturity	4 years	5 years	5 years	5 years	3 years	3 years
Covered Area	US 70%, UK 15%, F 7.5%, I 5%, CH 2.5%	US 62.5%, UK 17.5%, D 7.5%, J 7.5%, CAN 5%			US 100%	

Issued	Osiris Capital Plc. Nov. 2006				Vita Capital III Ltd. Dec. 2006. Jan. 2007			
Class	B1*	B2	C	D	A-IV*	A-V*	A-VI*	A-VII
Tranche Size	Euro 100mn	Euro 50mn	$150mn	$100mn	$100mn	$100mn	Euro 55mn	Euro 100mn
Arranger	Swiss Re				Swiss Re			
Protection for	AXA				Swiss Re			
Rating	Aaa/AAA	A3/A-	Baa2/BBB	Ba1/BB+	Aaa/AAA	Aaa/AAA	Aaa/AAA	Aa2/AA-
Attachment Point	114%	114%	110%	106%	125%	125%	125%	125%
Detachment Point	119%	119%	114%	110%	145%	145%	145%	145%
Coupon (bps)	EURIBOR+20	EURIBOR+120	LIBOR+285	LIBOR+500	LIBOR+21	LIBOR+20	EURIBOR+21	EURIBOR+80
Expected Maturity	4 years	4 years	4 years	4 years	4 years	5 years	4 years	5 years
Covered Area	F 60%, J 25%, US 15%				US 62.5%, UK 17.5%, D 7.5%, J 7.5%, CAN 5%			

Issued	Vita Capital III Ltd. (cont.) Dec. 2006. Jan. 2007					Nathan Ltd. Feb. 2008	SCOR Mortality Swap Jan. 2008
Class	B-I	B-II	B-III	BV*	BVI*	A	*na*
Tranche Size	$90mn	$50mn	Euro 30mn	$ 50mn	Euro 55mn	$100mn	$100mn + Euro 36mn
Arranger	Swiss Re					Munich Re, JPMorgan	JPMorgan
Protection for	Swiss Re					Munich Re	SCOR
Rating	A1/A	A1/A	A1/A	Aaa/AAA	Aaa/AAA	A2/A-	*na*
Attachment Point	120%	120%	120%	120%	120%	120%	115%
Detachment Point	125%	125%	125%	125%	125%	130%	125%
Coupon (bps)	LIBOR+110	LIBOR+112	EURIBOR+110	LIBOR+21	EURIBOR+22	LIBOR+135	*na*
Expected Maturity	4 years	5 years	4 years	5 years	4 years	5 years	4 year
Covered Area	US 62.5%, UK 17.5%, D 7.5%, J 7.5%, CAN 5%					US 45%, CAN 25%, UK 25%, D 5%	US, Europe

Table 3: Comparison of all CATM deals from 2003 until 2008 (Source: New Issue Reports from S&P and Moody's; *Bloomberg* data).

[5] The tranches marked with * are guaranteed by monoline insurers. Most of these tranches were downgraded in 2008 due to financial trouble of the guarantors.
[6] Rating at Issuance from Moody's / S&P – the ratings marked with ** were upgraded by S&P.

Structure of a CATM transaction

- Combined Mortality Index i_t contingent on the relative, weighted mortality experience of a certain population as reported from official entities in the years t and $t-1$:

$$\begin{aligned} i_t &= \frac{\frac{1}{2}\left(\hat{q}_t + \hat{q}_{t-1}\right)}{\frac{1}{2}\left(\hat{q}_{2005} + \hat{q}_{2004}\right)} \qquad \text{where} \\ \hat{q}_t &= \sum_{\text{all } x} \omega_{x,m}\, \hat{q}_{m,x,t} \;+\; \omega_{x,f}\, \hat{q}_{f,x,t} \end{aligned}$$

- Loss Tranche(a, d)

$$l_t^{(a,d)} = \min\left\{\max\left\{l_{t-1}, \frac{i_t - a}{d - a}\right\}, 100\%\right\}$$

with $l_{2006}^{(a,d)} := 0$, a *Attachment Point* (e.g. 110%), d *Detachment Point* (e.g. 120%).

Stochastic Mortality Modeling

- Array of stochastic mortality models available: Cairns et al. (2006, 2007,...)
- Affine stochastic mortality: Biffis (2005,IME), Dahl (2004,IME), Schrager (2006,IME)
- Here:
  - $\longrightarrow$ need jumps
  - $\longrightarrow$ want, parsimonious affine structure ...no suitable candidate
  - $\longrightarrow$ "coherent" specification

Stochastic Mortality Modeling

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- Here: $\longrightarrow$ need jumps
  $\longrightarrow$ want, parsimonious affine structure ...no suitable candidate
  $\longrightarrow$ "coherent" specification
- (Yet another) model:
  - Mean-reverting or not mean-reverting? Trend?

  $\Rightarrow$ Rely on demographic data and research:

  Positivity, Vaupel line (Oeppen & Vaupel (2002,Science)), "realist view on future longevity" (Olshansky, Carnes & Désquelles (2001, Science), Carnes & Olshansky (2007, Pop&DemRev)), rectangularization (Wilmoth & Horiuchi (1999, Demography)), mortality spikes with additive influence

Our model

$$\mu_t(x) = e^{b(x+t)}\, Y_t + \Gamma_t$$

where

$$dY_t = \underbrace{\alpha\left(\left(Y_0 - \beta^{(2)}\right) e^{-\beta^{(1)}\,t} + \beta^{(2)} - Y_t\right) dt}_{*} + \underbrace{\sigma\,\sqrt{Y_t}\,dW_t}_{\text{"positivity"}},$$

$$d\Gamma_t = \underbrace{-\kappa\,\Gamma_t\,dt + dJ_t}_{\text{"spikes"}},\ J_t \text{ CPP with Exp. distr. jumps}$$

Our model

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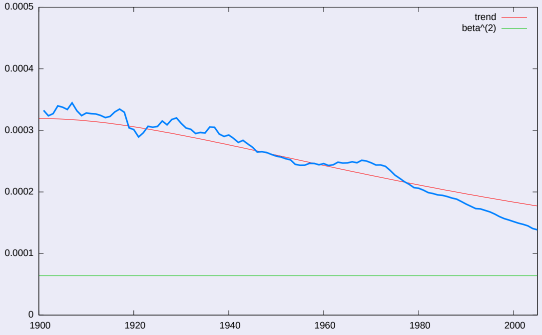
Solution to $*$ disregarding stochastic part (ODE)



- $t_{IP} \approx 44$ (inversion point)
- $\mathring{e}_\infty \approx 86$ years

$\longrightarrow$ in line with demographic research

Estimation

- Simulated Maximum Likelihood Estimation
- Particle Filter for Likelihood Evaluation
  ("Monte Carlo version of Kalman filter")
- Issues:
  - p_x's depend not only on μ_x, but on $[\mu_x, \mu_{x+1})$

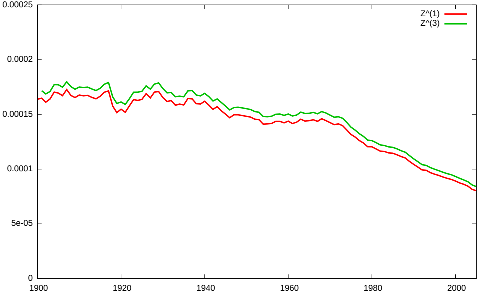
  $\longrightarrow$ Consider 4-dimensional state vector

$$\left(\underbrace{Y_t}_{Z_t^{(1)}}, \underbrace{\Gamma_t}_{Z_t^{(2)}}, \underbrace{\int_t^{t+1} e^{b(s-t)} Y_s \, ds}_{Z_t^{(3)}}, \underbrace{\int_t^{t+1} \Gamma_s \, ds}_{Z_t^{(4)}} \right)$$

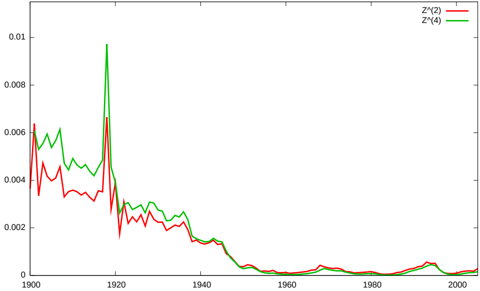
  - regularity of likelihood function, local maxima in optimization
    $\rightarrow$ Sorry, Dr. Brockett!

  $\longrightarrow$ used many many starting vectors based on smaller samples, final optimization with bigger sample
- Pros:
  - coherent ML estimation, model comparisons possible
  - "disentangle" jumps from continuous part, obtain distribution of states

Expected values of the states



- Allocation of initial improvements to jump component
- Clear jump event in 1918 $\rightarrow$ Spanish Flue
- Non-pandemic events noticeable (WWII or Vietnam war). Events affecting elderly population covered by cont. part



- Jumps necessary?

$\longrightarrow$ *p*-value of LR test essentially zero, Bayes factor $\exp\{1688.2\}$

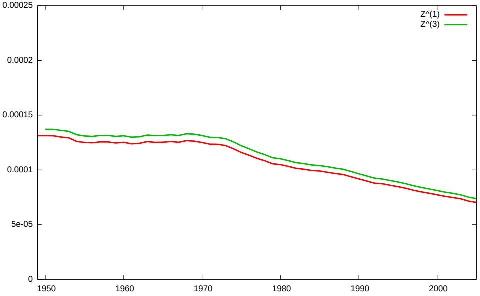
$\Rightarrow$ **Strong statistical evidence**

Loss Profiles (Tartan deal)

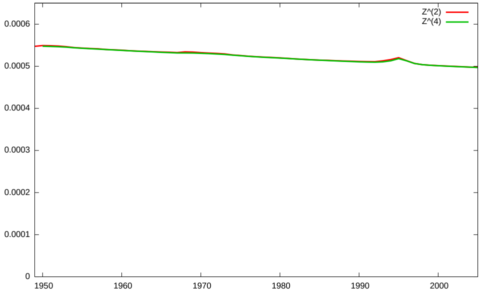
	PD(%)	EL(%)	Spread(bps)
Cl. B Tranche(110%-115%)			
Jump Model, data 1901-2005	18.08	16.60	693
Model without jumps, data 1901-2005	9.69	3.35	111
Reported	0.88	0.54	–
Cl. A Tranche(115%-120%)			
Jump Model, data 1901-2005	15.24	14.04	582
Model without jumps, data 1901-2005	0.47	0.12	4
Reported	0.29	0.16	–

- Risk measures exceed official profiles
- "Actuarially fair" spread for jump model exceeds market spread (300bps for Tranche B)

Estimation results based on 1950-2005 data



- lower volatility
- limiting life expectancy about the same



- basically no jumps

Loss Profiles (Tartan deal)

	PD(%)	EL(%)	Spread(bps)
Cl. B Tranche(110%-115%)			
Jump Model, data 1901-2005	18.08	16.60	693
Model without jumps, data 1901-2005	9.69	3.35	111
Jump Model, data 1950-2005	0.89	0.80	32
Reported	0.88	0.54	–
Cl. A Tranche(115%-120%)			
Jump Model, data 1901-2005	15.24	14.04	582
Model without jumps, data 1901-2005	0.47	0.12	4
Jump Model, data 1950-2005	0.71	0.64	26
Reported	0.29	0.16	–

- Risk characteristics strongly depend on estimation period
- Investors' beliefs in line with permanent regime change

"Endogenous" valuation

- Idea: (cf. Lin & Cox (2005,JRI), Bauer, Börger & Ruß (2009,IME))
  - Derive risk-adjusted parametrization based on primary insurance prices – yields at least an upper bound for mortality derivatives

  $\longrightarrow$ Fast calibration due to affine structure, results based on 73 term-life quotes

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Baseline Component

α	$\beta^{(1)}$	$\beta^{(2)}$	σ^*	b^*	Y_0^*
0.018 $\rightarrow$ **4.695**	0.0288 $\rightarrow$ –	0.00006 $\rightarrow$ **0.00003**	0.00026	0.083	0.00016

Catastrophe Component

κ	λ	ζ	Γ_0^*
1.033 $\rightarrow$ **3.24E-11**	0.114 $\rightarrow$ **1792**	211 $\rightarrow$ **1.49E+07**	0.0036

- **What happened?**
  - lower baseline mortality: differences in populations, selection effects
  - permanent, small, high-frequency jumps: selection effects – approximation to ultimate mortality

$\longrightarrow$ These effects overshadow potential mortality risk premium. Risk measures basically zero.

Conclusion

- Explain structure of CATM securitization, provide market overview
- Present time-continuous stochastic mortality model for the analysis of CATM bonds. With "only" eight parameters, our model...
  - ... displays basic features that are in line with demographic data and research
  - ... shows jumps that are structurally consistent with catastrophic mortality events observed in the last century
  - ... offers a high degree of analytical tractability due to affine structure

- Primary result:

  Calculated risk profiles significantly exceed official loss profiles for most calibrations, large uncertainties $\Rightarrow$ loss profiles should be interpreted very carefully by investors and rating agencies

- Future research:
  - extend model to multiple populations
  - can uncertainty aversion (ambiguity aversion) explain observed spread levels?

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Thank you!

Extra 1: Short calibration trend component