Longevity Five 2009

The Poisson log-bilinear Lee Carter model: Efficient bootstrap in life annuity actuarial analysis

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Background and Contribution

- **Longevity risk** of crucial importance for annuity providers due to **long contract terms**; but also **investment risk** matters
- **Risk quantification** important for e.g. **solvency** analyses (cf. Solvency II: internal models), focus on **asset-liability-ratio**
- **Lee-Carter** model is de facto **standard** in industry and academia, but has several drawbacks "solved" by **extension and variations**
  - popular: Poisson assumption (Brouhns et al.; Renshaw/Haberman)
- **Precision** of mortality forecasts to be **quantified** (confidence intervals) and **improved** (variance reduction techniques)
  - Bootstrap
  - Stratified sampling
- Present paper: assessment of **longevity** and **interest rates risk** and the **impact on** life annuity **funding ratios**

Summary and Structure

- **Funding ratio**: MV(future assets) / MV(future liabilities) wrt portfolio
  - Homogeneous portfolio of 1000 constant premium deferred annuities
  - Subject to mortality and interest rates
  - Inception age 30, lifelong payouts from age 65 on
- **Mortality** based on Lee-Carter, extended by **Poisson assumption**
  - Three **variance reduction techniques (VRT)** paralleled:
    - SP: Standard Procedure (semi-parametric bootstrap; Brouhns et al.)
    - IP: Iterative Procedure (Renshaw/Haberman)
    - SSP: Stratified Sampling (to reduce population heterogeneity)
  - Calibrated to Italian male data (1950-2006)
- **Interest rate** modeled as Heath/Jarrow/Morton
  - Calibrated to EURIBOR quarterly rates (01/2002-03/2009)
- **Premiums** calculated at 4% constant interest rate

Key Results – Interpretation and Caveat

- **Premiums**: **S**tandard**P**roc < **I**terative**P**roc < **S**tratified**S**ampling**P**roc
  Reflects increasing conservativeness, but stronger degree of variance reduction should result in lower premiums – which is preferable?
- **Funding ratios**
  - **SP** < **IP** < **SSP** for **all** time horizons. Clearly due to premium order.
  - Strongly **positive** – between 1.13 and 1.58 (or 1.86).
    Suggests excessive premiums given actual mortality/interest rates – couldn't (net) premiums be lowered (or benefits increased)?
  - Strictly **increasing over time.**
    More than sufficient premiums during accumulation w/o payouts lead to ≈50% excess funding – NO profit sharing. Excess amount not needed to fulfill obligations during payout phase, instead further accumulation.
- **Projection risk:** variance of conditional expectation; SP/IP/SSP with probabilities 0.1/0.3/0.6; slight increase, reduced during payout phase. Probabilities arbitrary? Interpretation of values – small or large?

Suggestions for Improvement

- Comment on the 4% **constant interest rate versus HJM** predictions.
  - Is a 4% fixed interest rate adequate for premium calculation?
  - What (higher) rate (or lower premiums) would still be sufficient?
- Comment on the **high degree of overfunding**.
  - Unbounded increase of funding ratio may seem preferable for insurers, but likely to be problematic in terms of e.g. marketing.
  - What are the implications for risk management?
- Illustrate **Stratified Sampling Procedure, determination of strata**.
- Minor suggestions and remarks:
  - Check dimension of projection risk – is it "only" 0.06% to 0.35%?
  - Reorder diagrams: SP/IP/SSP would be consistent with tables.
  - Check notation of variables: time horizon: ***r*** vs. ***t*** vs. ***T***; discount: ***w(t,j)***.
  - Introduce the symbols used in Eq. (7) – (13).