

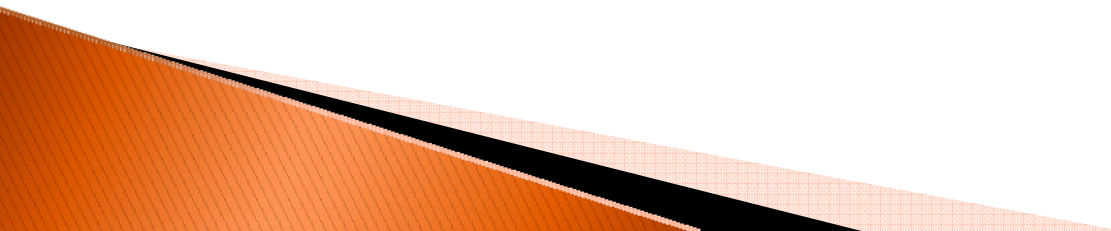
Hedging Longevity Risk by Asset Management: an ALM Approach

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Introduction

- ▶ This paper intends to
 - propose an asset/liability management approach to control the aggregate risk for annuity providers or pension funds
 - develop an integrated model which can deal with interest rate risk and longevity risk simultaneously
 - demonstrate that our proposed approach can lead to an optimal asset liability allocation strategy which can effectively decrease the aggregate risks of annuity providers
 - longevity bond can be served as an efficient vehicle to hedge against longevity risk
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Introduction

- ▶ An unprecedented improvement in population longevity has occurred globally over the course of the twentieth century
- ▶ Longevity risk represents a critical threat to pension funds and private insurers
 - increases the payout period and the liability costs of providing annuities
- ▶ Hedging longevity risk has received serious attention
 - mispricing on annuity products or misallocation of investment strategies could cause substantial deficits

Introduction---Literature Review

The literature of longevity risk has contributed in suggesting:

a better models to predict mortality rates

Adapt the discrete-time models

- Lee and Carter (1992), Renshaw and Haberman (2003), and Cairns et al. (2006, 2007)

In a continuous-time framework

- Dahl (2004), Schrager (2006), Ballotta and Haberman (2006), and Hainaut and Devolder (2008)

a better strategies to hedge the longevity risk

Employ mortality securitization

- Dowd (2003), Blake et al. (2006a, 2006b), Lin and Cox (2005) and Cox et al. (2006)

Use survivor bonds and survivor swap

- Blake and Burrows (2001), Denuit, Devolder and Goderniaux (2007) and Dowd et al. (2006)

Use a nature hedging strategy

- Cox and Lin (2007) and Wang et al. (2008 and 2009)

Use a reinsurance swap

- Lin and Cox (2005)

Introduction

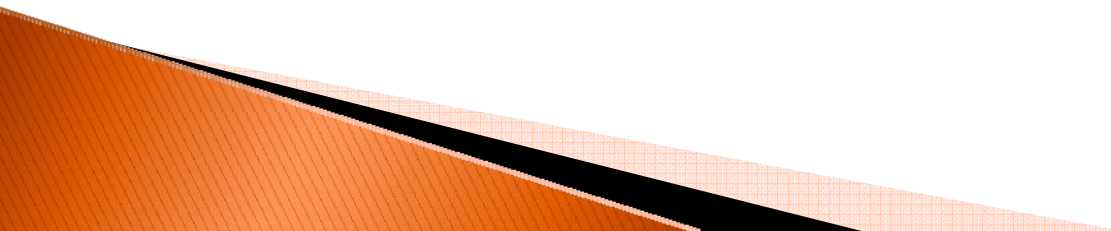
- ▶ Although the literature has provided many ingenious strategies to hedge longevity risk, most papers analyzed the effects of their strategies under a common assumption that the insurance company faces longevity risk but did not consider the interest rate risk at the same time.
- ▶ Ignoring interest rate risk may results in underestimating the aggregate risk and misleading the hedging strategy.
- ▶ In this paper, we develop an integrated model which can deal with interest rate risk and longevity risk simultaneously.

Introduction

▶ Assumption:

- annuity providers could invest in longevity bond, coupon bond and risk-free bond to minimize the risks of equity holders subject to a targeted profit

▶ Contributions of our paper:

- propose a more realistic hedging strategy for the annuity providers by considering both interest rate risk and longevity risk simultaneously
 - find that longevity bond can be served as an effective vehicle in the asset/liability management to significantly reduce the aggregate risk for annuity providers
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The CIR model

- ▶ If the interest rate follows the stochastic process suggested by Cox, Ingersoll, and Ross (1985), then the interest rate path can be expressed as

$$dr_t = a(b - r_t)dt + \sigma \sqrt{r_t} dz \quad (1)$$

- ▶ where a , b , and σ are constants, dz follows a standard Brownian motion
- ▶ The drift rate of the interest rate is $a(b - r_t)$
- ▶ The standard deviation of the interest rate is $\sigma \sqrt{r_t}$
- ▶ CIR assumed that the variation in the interest rate in each period is proportional to the square root of the interest rate in each period

The CIR model

- ▶ Cox, Ingersoll, and Ross (1985) solved equation and showed that

$$P_t = \alpha_t e^{-\beta_t r}$$

- ▶ where P_t is the current price of a one-dollar zero-coupon bond of periods and

$$\alpha_t = \left[\frac{2\sqrt{a^2 + 2\sigma^2} e^{\frac{t}{2}(a + \sqrt{a^2 + 2\sigma^2})}}{(\sqrt{a^2 + 2\sigma^2} + a^2)(e^{t\sqrt{a^2 + 2\sigma^2}} - 1) + 2\sqrt{a^2 + 2\sigma^2}} \right]^{2ab/\sigma^2},$$
$$\beta_t = \frac{2(e^{ta + \sqrt{a^2 + 2\sigma^2}} - 1)}{(\sqrt{a^2 + 2\sigma^2} + a^2)(e^{t\sqrt{a^2 + 2\sigma^2}} - 1) + 2\sqrt{a^2 + 2\sigma^2}}. \quad (2)$$

The Two-Factor Stochastic Mortality Model

- ▶ We choose the two-factor mortality model (the CBD model) as the underlying mortality process
- ▶ Let $q_{t,x}$ be the realized mortality rate for age x insured from time t to $t+1$. Assume the mortality curve has a logistic functional form:

$$q_{t,x} = \frac{e^{A_1(t+1)+A_2(t+1)\cdot(x+t)}}{1 + e^{A_1(t+1)+A_2(t+1)\cdot(x+t)}}. \quad (3)$$

- ▶ $A_1(t)$ and $A_2(t)$ in the model represent all age-general improvements in mortality over time (trend effect) and different improvements for different age groups (age effect)

The Two-Factor Stochastic Mortality Model

- ▶ The two stochastic trends $A_1(t+1)$ and $A_2(t+1)$ follow a random walk process with a drift parameter μ and a diffusion parameter C :

$$A(t+1) = A(t) + \mu + CZ(t+1), \quad (4)$$

- ▶ Where $A(t+1) = [A_1(t+1), A_2(t+1)]^T$ and $\mu = [\mu_1, \mu_2]^T$ are a 2×1 constant parameter vector
- ▶ C is a 2×2 constant upper-triangular Cholesky square root matrix of the covariance matrix $V = CC^T$ and $Z(t)$ is a two-dimensional standard normal random variable

The Two-Factor Stochastic Mortality Model

- ▶ To include the uncertainty of μ and C , Cairns et al. (2006b) invoke a normal-inverse-Wishart distribution from a non-informative prior distribution:

$$V^{-1} | D \sim \text{Wishart}(n - 1, n^{-1} \hat{V}^{-1}) \quad (5)$$
$$\mu^{-1} | V, D \sim \text{MVN}(\hat{\mu}, n^{-1} V),$$

where $D(t) = A(t) - A(t - 1),$

$$\hat{\mu} = \frac{1}{n} \sum_{t=1}^n D(t),$$

and $\hat{V} = \frac{1}{n} \sum_{t=1}^n (D(t) - \hat{\mu})(D(t) - \hat{\mu})^T.$

The Hedging Approach

- ▶ This paper develops the asset liability strategy in a Mean-Variance framework
- ▶ Assume:
 - a annuity provider receives A dollar annuity premium
 - invest the money in coupon bond B , longevity bond B^l and risk free bond B^{rf}
- ▶ The unexpected profit or loss for the equity holders at next rebalance day will be

$$E = xB(r) + yB^l(r, m) + zB^{rf}(r) - A(r, m) \quad (6)$$

The Hedging Approach

$$E = xB(r) + yB^l(r, m) + zB^{rf}(r) - A(r, m)$$

- ▶ $A(r, m)$ - Total value of annuity position, which depends on interest rate r and mortality rate m .
- ▶ $x \cdot B(r)$ - Total value of long term Treasury bond
- ▶ $y \cdot B^l(r, m)$ - Total value of longevity bond.
- ▶ $z \cdot B^{rf}(r)$ - Total value of short term risk-free bond

The Hedging Approach

- ▶ Applying Taylor's expansion on the change of Equity with respect to r and m , gives us

$$\Delta E \approx \delta_r^E \Delta r + \delta_m^E \Delta m + \frac{1}{2} \gamma_r^E \Delta r^2 + \frac{1}{2} \gamma_m^E \Delta m^2 + \gamma_{r,m}^E \Delta r \Delta m \quad (7)$$

- ▶ Δr and Δm are changes of interest rate and mortality rate
- ▶ δ_r^E and δ_m^E are the partial derivatives of E with respect to r and m
- ▶ γ_r^E and γ_m^E are the second partial derivatives E with respect to r and m
- ▶ $\gamma_{r,m}^E$ is the cross derivative with respect to r and m

The Hedging Approach

- ▶ Substituting equation (6) into equation (7) gives us

$$\begin{aligned}\Delta E = & \left(x\delta_r^B + y\delta_r^{B^l} + z\delta_r^{B^{rf}} - \delta_r^A\right)\Delta r + (y\delta_m^{B^l} - \delta_m^A)\Delta m + \frac{1}{2}\left(x\gamma_r^B + y\gamma_r^{B^l} + z\gamma_r^{B^{rf}} - \gamma_r^A\right)\Delta r^2 \\ & + \frac{1}{2}(y\gamma_m^{B^l} - \gamma_m^A)\Delta m^2 + (y\gamma_{r,m}^{B^l} - \gamma_{r,m}^A)\Delta r\Delta m\end{aligned}\quad (8)$$

- ▶ $\delta^B, \delta^{B^l}, \delta^{B^{rf}}$ and δ^A are partial derivatives for the assets and liabilities with respect to r or m
- ▶ $\gamma^B, \gamma^{B^l}, \gamma^{B^{rf}}$ and γ^A are second partial derivatives for the assets and liabilities with respect to r or m
- ▶ $\gamma_{r,m}^{B^l}$ and $\gamma_{r,m}^A$ are cross derivative with respect to r and m

The Hedging Approach

- ▶ The variance of ΔE is

$$V(\Delta E) = \beta_1 x^2 + \beta_2 y^2 + \beta_3 z^2 + \beta_4 xy + \beta_5 xz + \beta_6 yz + \beta_7 x + \beta_8 y + \beta_9 z + \beta_{10} \quad (10)$$

- ▶ $\beta_1, \beta_2, \dots, \beta_{10}$ are coefficient of derivatives ($\delta_r, \delta_m, \gamma_r, \gamma_m, \gamma_{r,m}$) and moments ($\sigma_r^2, E(\Delta r^3), E(\Delta m^3)$)
- ▶ They are all independent of investment volume x, y , and z

The Hedging Approach

- ▶ We could choose the investment amounts x , y , and z to minimize $V(\Delta E)$ subject to the profit constraint:

$$\text{Min}_{x, y, z} \quad V(\Delta E) \quad (11)$$

$$s.t. \quad E(\Delta E) + xBR^B + yB^lR^{B^l} + zB^{rf}R_f - AR_a \geq R_S S \quad (12)$$

$$xB + yB^l + zB^{rf} = A + S \quad (13)$$

- ▶ $V(\Delta E)$ is the variance of the change value in equity
- ▶ $xBR^B + yB^lR^{B^l} + zB^{rf}R_f - AR_a$ is the interest or investment earned from contracts which is independent of Δr and Δm
- ▶ R_S is the required return for equity holders for total profit
- ▶ S is the required surplus corresponding to A

Numerical Examples

▶ Assumption:

- an annuity provider could invest in longevity bond, coupon bond and risk-free bond to minimize the risks of equity holders subject to a targeted profit

▶ Longevity bond

- Invest the Bank BNP Paribas and the European Investment Bank (EIB) launched a 540 million bond with a 25-year maturity longevity bond
- The bond's cash flows will be based on the actual longevity experience of the English and Welsh male population aged 65 years old
- The future cash flows of the bond will be equal to the amount of a fixed annuity multiplied by the percentage of the reference population still alive at each time point
- We use the CBD model of Cairns et al. (2006) to pricing the longevity bond

Numerical Examples

Table 1 The CBD mortality rate model parameters.

λ_1	market price of longevity risk associated with level shift in mortality	0.175
λ_2	market price of longevity risk associated with tilt in mortality	0.175
X	initial age of cohort	65
T	Bond Maturity	25

- The value of longevity bond on issuing day is 11.442

Numerical Examples

- ▶ We use the CIR interest model to estimate the interest rate yield curve
- ▶ The values are taken from Ahlgrim et al. (2004)
- ▶ The initial interest rate is the yield on the shortest maturity U.S. Treasury strip (August 2001) as of June 8, 2001.

Table 2 The CIR interest rate model parameters.

a	speed of mean reversion	0.25
b	long run mean interest rate	0.066
σ	volatility	0.08
r	initial interest rate	0.0362

Numerical Examples

▶ Assets:

- Treasury bond is issued at 5% coupon rate, 30-years, face value 1,000 and the price is 1,250
- The LB is issued at LIBOR-35bps coupon rate, 25-years, face value 10,000 and yield is 6.25%
- Treasury bill is pure discount bond and sell at 965.06

▶ Liability:

- The whole-life annuity is issued for age-60 men and pay the cohort groups US \$1,000 at the end of each year
- The premiums are collected in a single premium today
- Assume there is no deferred period
- The mortality process follows the Cairns-Blake-Dowd two-factor model (CBD model).

The Results

Table 4 Delta and gamma measures parameters.

	Interest rate sensitivity	Mortality rate sensitivity
delta	$\delta_r^B = -16.846$	
	$\delta_r^{B^l} = -9.773$	$\delta_m^{B^l} = -2.311$
	$\delta_r^{B^{rf}} = -0.971$	
	$\delta_r^A = -11.256$	$\delta_m^A = -2.971$
gamma	$\gamma_r^B = 197.722$	
	$\gamma_r^{B^l} = 145.778$	$\gamma_m^{B^l} = 0.001$
	$\gamma_r^{B^{rf}} = 0.943$	
	$\gamma_r^A = 202.711$	$\gamma_m^A = 0.003$
cross gamma	$\gamma_{r,m}^{B^l} = -5.028$	
	$\gamma_{r,m}^A = -5.966$	

The Results

Table 5 Optimal allocations value under different required rate of return

Case	$(\Delta r, \Delta m, R_s)$	% of B	% of B^l	% of B^{rf}	aggregate risk of equity
Case A:	(2.5%, 1.10, 6%)	0.6237	–	0.3763	101.57
without investment in LB	(1.5%, 1.10, 6%)	0.6062	–	0.3938	108.50
	(1.0%, 1.20, 6%)	0.6230	–	0.3770	160.11
Case B:	(1.0%, 1.10, 6%)	0.1701	0.5254	0.3045	43.41
with investment in LB	(1.5%, 1.10, 6%)	0.1029	0.5870	0.3101	46.79
	(1.0%, 1.20, 6%)	0.0487	0.6508	0.3005	69.26
Case C:	(1.0%, 1.10, 8%)	0.2501	0.5954	0.1545	80.47
require higher R_s	(1.5%, 1.10, 8%)	0.2291	0.6044	0.1665	89.55
	(1.0%, 1.20, 8%)	0.1287	0.7208	0.1505	107.90

The Results

- ▶ The numerical results show that the optimal ALM strategies suggest to increase the investment in longevity bond with a significant proportion 0.5254 (more than 50%) in Case B.
- ▶ After hedging with longevity bond, the aggregate risk of equity is significantly reduced from 101.57 to 43.41.
- ▶ In Case C, we raise the required profit returns from 6% to 8%. The optimal ALM allocations suggest to increase the investments in Treasury bond and longevity bond and reduce the investment in short-term risk free bond.
- ▶ The higher profit is accomplished with higher aggregate risk of equity.

The Results

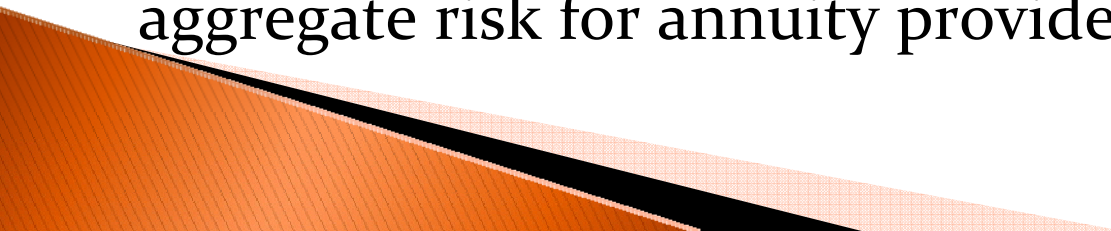
Table 6 Optimal allocations value under different CIR parameters

Case	(a, b, σ)	% of B	% of B^l	% of B^{rf}	aggregate risk of equity
Base case	(0.25, 6.6% , 0.08)	0.1029	0.5870	0.3101	46.79
Case (1)	(0.10, 6.6% , 0.08)	0.0602	0.5628	0.3770	58.60
Case (2)	(0.40, 6.6% , 0.08)	0.2014	0.6582	0.1404	43.36
Case (3)	(0.25, 5.0% , 0.08)	0.0881	0.5254	0.3865	49.81
Case (4)	(0.25, 8.0% , 0.08)	0.1964	0.6050	0.1976	43.66
Case (5)	(0.25, 6.6% , 0.06)	0.1108	0.6102	0.2790	39.55
Case (6)	(0.25, 6.6% , 0.10)	0.0923	0.5540	0.3537	61.40

The Results

- ▶ In Case (1) and (2), we consider the change in mean reversion speed. As the mean reversion increase, ALM suggest to increase the investments in Treasury bond and longevity bond and reduce the investment in short-term risk free bond.
- ▶ In Case (3) and (4), we consider change in long-term interest rate. As the long-term interest rate increase, we still increase the investments in Treasury bond and longevity bond and reduce the investment in short-term risk free bond.
- ▶ In Case (5) and (6), we consider the change in interest rate volatility. As the mean reversion increase, we decrease the investments in Treasury bond and longevity bond and increase the investment in short-term risk free bond.

Conclusion

- ▶ This paper proposes an asset liability management (ALM) strategy to hedge the aggregate risk of Equity for annuity providers by considering both stochastic interest rate risk and mortality rate risk simultaneously.
 - ▶ The numerical examples shows that, failing to integrate interest rate risk into the model for hedging longevity risk could cause a substantial risk for equity holders.
 - ▶ The simulation results also suggest that longevity bond plays a critical role in the integrated model.
 - ▶ We demonstrate that our proposed approach can lead to an optimal asset liability allocation and longevity bond can be served as an effective vehicle to significantly reduce the aggregate risk for annuity providers.
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Thank you