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# The Impact of Natural Hedging on a Life Insurer's Risk Situation

Longevity 7
September 2011

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# Introduction
## Motivation

- Demographic risk can significantly impact a life insurer's solvency level
  - ➢ Increase in life expectancy poses serious problems to life insurers selling annuities
  - ➢ However, risk of unexpected high mortality (e.g. due to pandemics) has increased as well; problem for term life
- But: Hedging instruments are still scarce
  - ➢ "Natural Hedge" between term life insurance (death benefit) and annuities (lifelong survival benefits) is effective alternative
  - ➢ Use opposed reaction of term life insurance and annuities towards shocks to mortality
  - ➢ Hedge shocks to mortality internally through portfolio composition

# Introduction
## Aim of paper

- Previous literature:
  - Cox/Lin (2007), Bayraktar/Young (2007), Gründl/Post/Schulze (2006), Wang et al. (2010), Wetzel/Zwiesler (2008)

- Aim of this paper:

1. Quantify impact of natural hedging on a life insurance company's insolvency risk
   - Holistic model, take into account dynamic interaction between assets and liabilities for a two-product life insurer
2. Simultaneously *immunize* an insurer's solvency situation against changes in mortality and *fix the absolute level of risk*
   - Use investment strategy

# Model framework
## Modeling and forecasting mortality

- Extension of the Lee-Carter (1992) model by Brouhns/Denuit/ Vermunt (2002):

$$D_{x,t} \sim Poisson\left(E_{x,t} \cdot \mu_x(t)\right) \quad \mu_x(t) = \exp\left(a_x + b_x \cdot k_t\right) \quad q_x(t) = 1 - \exp\left(-\mu_x(t)\right)$$

  - $D_{x,t}$ Poisson-distributed number of deaths, $E_{x,t}$ exposure at risk
  - $a_x$ and $b_x$ indicating the general shape of mortality over age
  - $k_t$ indicating the general level of mortality in the population (with negative drift)
  - Forecasting of $k_t$ (and $\mu_x(t)$) by ARIMA process for estimated time series of $k_t$

# Model framework
## Modeling systematic mortality risk

- Analyze systematic mortality risk in two ways:

1. Shock to (decreasing) mortality time trend: $e^*k_t$
   - Leads to an unexpected change in the level and future development of mortality
   - Shocks e > 1: mortality rates decrease (longevity scenario)
   - Shocks e < 1: mortality rates increase (pandemic scenario)
   - How to compose a portfolio of term life and annuities in order to immunize the portfolio against shocks to mortality?

2. Use empirically observed changes in mortality
   - Analyze usefulness of natural hedging under realized changes in mortality
   - Similar results

# Model framework
## Model of a life insurance company

- Simplified balance sheet:

| Assets | Liabilities |
|---|---|
| $A(t)$ | $E(t)$ |
| | $B_A(t)$ ⎫ $L(t)$ |
| | $B_L(t)$ ⎭ |

  - o $A(t)$ : market value of assets at time $t$
  - o $B_A(t)$ : book value of liabilities for annuities at time $t$
  - o $B_L(t)$ : book value of liabilities for term life insurance at time $t$
  - o $E(t)$ : equity at time $t$

➢ Default of the insurance company, if $L(t) = B_L(t) + B_A(t) > A(t)$

# Model framework

## Liabilities – Premium and benefit calculation

- Premiums and benefits: use actuarial equivalence principle
  - Term life insurance

$$\sum_{t=0}^{T-1} P \cdot {}_t p_x \cdot (1+r)^{-t} = \sum_{t=0}^{T-1} DB \cdot {}_t p_x \cdot q_{x+t} \cdot (1+r)^{-(t+1)}$$

  - Life-long immediate annuity

$$SP = \sum_{t=0}^{T-1} a \cdot {}_t p_x \cdot (1+r)^{-(t+1)}$$

- Improve comparability and isolate effect of natural hedging:
  - Calibrate input parameters such that volume of both contract types is identical at inception
  - Fix the number of contracts sold

# Model framework

## Liabilities – Book value of liabilities

- Use actuarial reserve to determine book value of liabilities
- Value of one term life insurance contract:

$$B_L(t)=\sum_{s=0}^{T-t-1}\left[DB\cdot{}_s\,p_{x+t}(e)\cdot q_{s+x+t}(e)\cdot(1+i)^{-(s+1)}-P\cdot{}_s\,p_{x+t}(e)\cdot(1+i)^{-s}\right]$$

- Value of one annuity:

$$B_A(t)=\sum_{s=0}^{T-t-1}a\cdot{}_s\,p_{x+t}(e)\cdot(1+i)^{-(s+1)}$$

- ➢ Mortality rates are subject to shock e
- ➢ Value of liabilities *L*(*t*):

$$L(t)=n_A(t)\cdot B_A(t)+n_L(t)\cdot B_L(t)$$

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# Model framework
## Assets

- Assets follow a geometric Brownian motion:

$$dA(t) = \mu \cdot A(t) \cdot dt + \sigma \cdot A(t) \cdot dW^{P}(t)$$

- Development of asset base depends on cash-flows of insurance portfolio

$t = 0^{+}$ | $t = 1^{-}$ | $t = 1^{+}$ | $t = 2^{-}$ | …

**$t = 0^{+}$:**
$+ E_0$
$+ n_A(0) \cdot SP$
$+ n_L(0) \cdot P$

**$t = 1^{-}$:**
$- n_A(1) \cdot a$
$- d_L(0) \cdot DB$
$- div$

**$t = 1^{+}$:**
$+ n_L(1) \cdot P$

**$t = 2^{-}$:**
$- n_A(2) \cdot a$
$- d_L(1) \cdot DB$
$- div$

…

Number of annuity contracts active in $t$ = 1

Constant dividend to shareholders

Number of life insurance contracts active in $t$ = 1

Number of annuity contracts active in $t$ = 2

Number of life insurance policyholders who died during $t$ = 0

Constant dividend to shareholders

Number of life insurance policyholders who died during $t$ = 1

# Model framework

## Risk measurement

- Probability of default (PD): $PD = P\left(T_d \leq T\right)$

  with $T_d = (T+1) \vee \inf\left\{t : A(t) < L(t)\right\}, t = 1,\ldots,T.$

- Mean Loss (ML): $ML = E\left(\max\left(\left(L(T_d) - A(T_d)\right)\cdot(1+r)^{-T_d}, 0\right)\cdot 1\{T_d \leq T\}\right)$

- Expected Shortfall (ES) $ES = \frac{ML}{PD}$

- Contractual Payment Obligations (CP)

$$CP = n_L(0)\cdot\sum_{t=0}^{T-1} DB\cdot {}_t p_x(e)\cdot q_{x+t}(e)\cdot(1+r)^{-(t+1)} + n_A(0)\cdot\sum_{t=0}^{T-1} a\cdot {}_t p_x(e)\cdot(1+r)^{-(t+1)}$$

  - Only liability side
  - Linear in portfolio composition

# Numerical results
## Input parameters

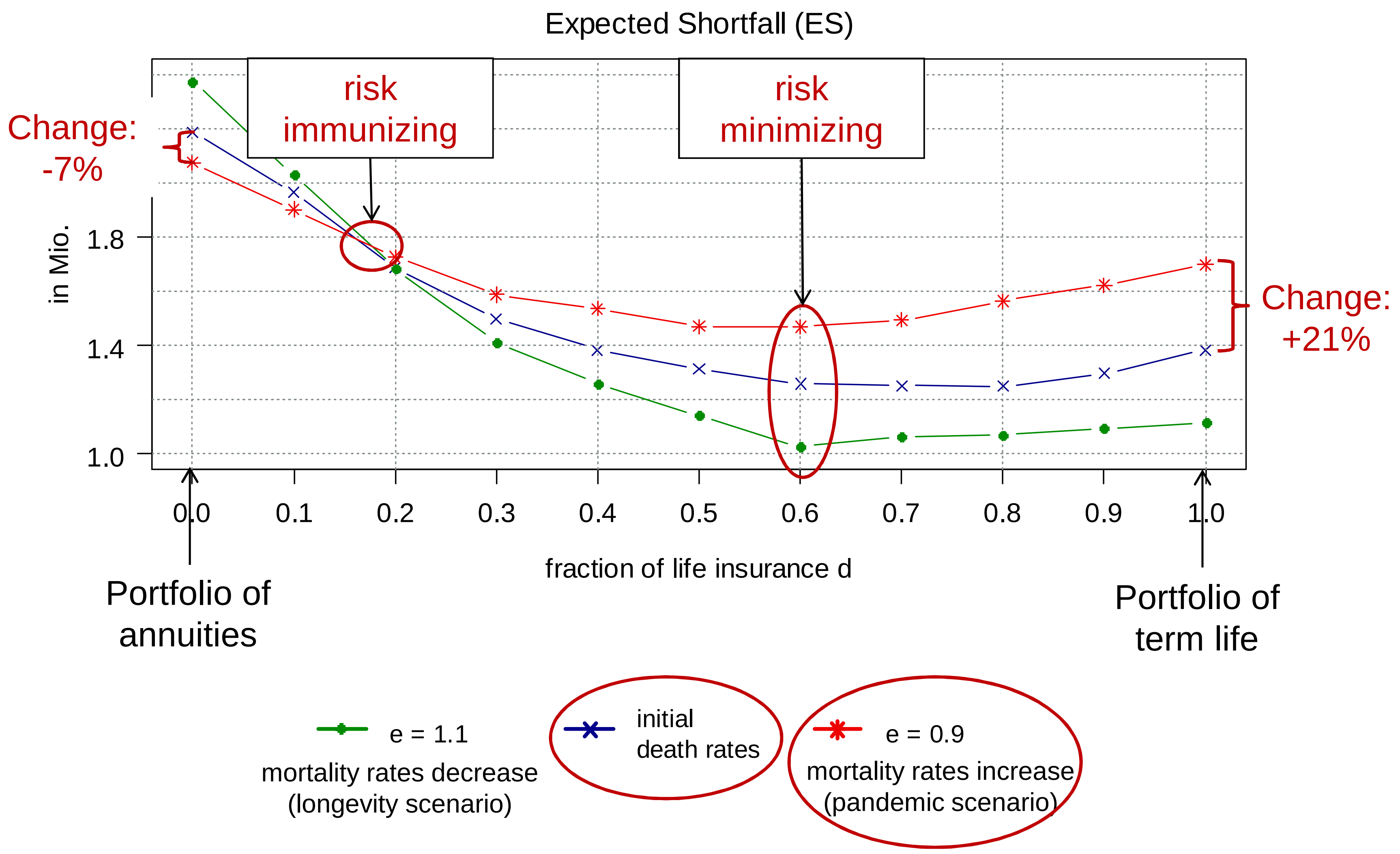
- Liabilities

| | |
|---|---|
| Age at inception of term life | 30 |
| Max. duration of term life | 35 |
| Age at inception of annuity | 65 |
| Premium for life insurance (*P*) | 417 |
| Single premium for annuity (*SP*) | 10,000 |
| Yearly annuity (*a*) | 725 |
| Death benefit (*DB*) | 88,724 |
| Total number of contracts sold | 10,000 |

- Assets

| | |
|---|---|
| Drift of assets ($\mu$) | 6% |
| Volatility of assets ($\sigma$) | 10% |
| Risk-free interest rate ($r$) | 3% |

# Numerical results
## Risk under different shocks to mortality



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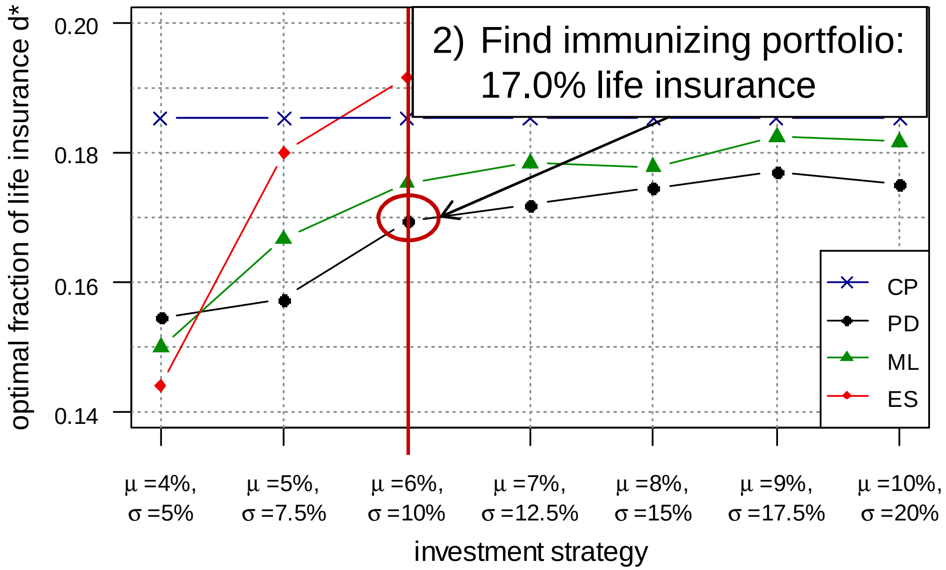
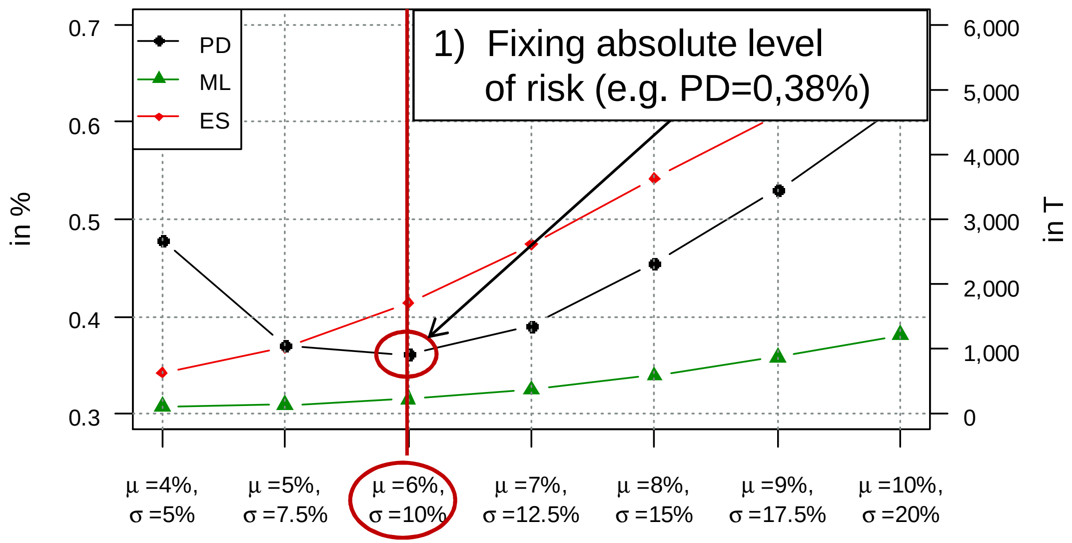
# Numerical results
## Varying the investment strategy




Optimal hedge ratio for different investment strategies




Corresponding level of insurer‘s default risk for optimal hedge ratio

Here: for a shock to mortality of *e* = 1.1 (longevity scenario)

# Summary

- Results show: Natural hedging can considerably reduce absolute risk level of an insurer and immunize it against shocks to mortality
  - Optimal portfolio composition depends on risk measure
  - Holistic consideration of mortality risk with respect to insurer's overall risk level is vital (focus on liability side only underestimates risk)
- Investment strategy can have substantial impact on the effectiveness of natural hedging
  - Use investment strategy to simultaneously *fix a risk level* and *immunize* the portfolio against shocks to mortality
  - Changing the investment strategy requires adjustment of portfolio mix to immunize portfolio against changes in mortality

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*Thank you very much for your attention!*

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