Regular Discount Sequence and Longevity Bond

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Outline

- Motivation
- Mortality Model and Regular Discount Sequences
- Prediction Method and Forecasting the Mortality Rates
- Pricing on longevity bond
- Conclusion and Discussion

Motivation

●The life expectancies in Taiwan are expected to continue increasing in a steady pace.

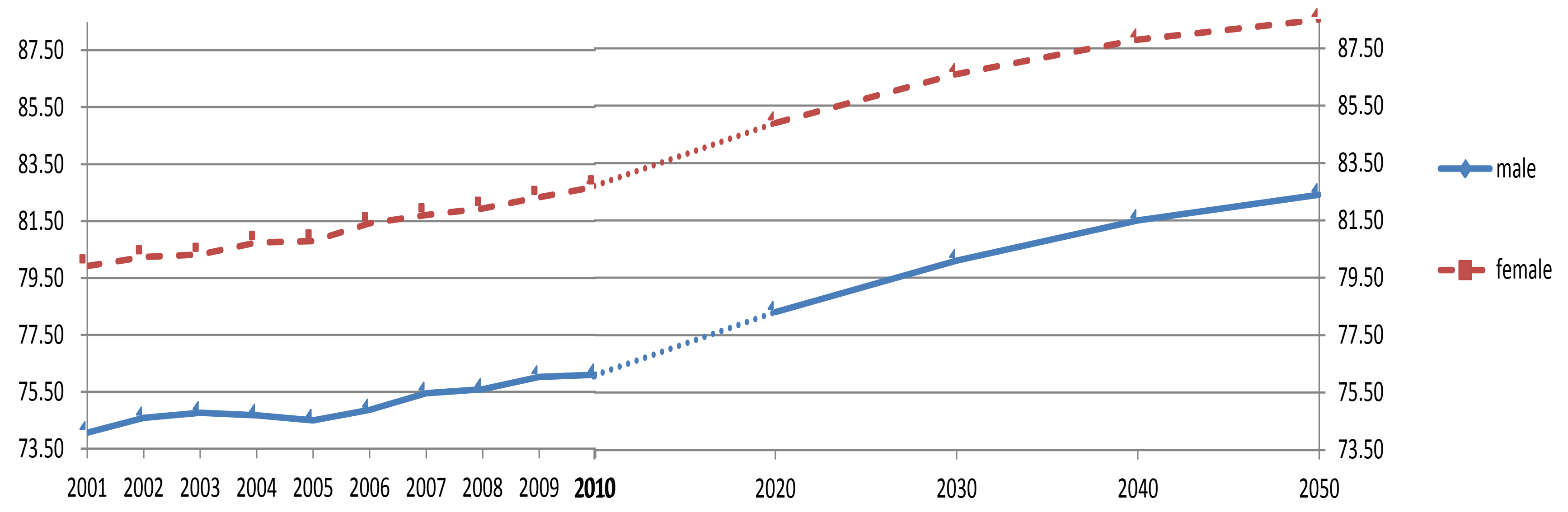
•Taiwan's life expectancies at birth :

Male	74.07(2001)➔76.1(2010)*➔82.4(2050)
Female	79.92(2001)➔82.7(2010)*➔88.5(2050)

*projection by Taiwan's Council for Economic Planning and Development.)

Motivation

- Taiwan's life expectancies at birth:



Motivation

- Rapid decreases in fertility and mortality in Taiwan:
- Percentages of population by age over 65 years has been increasing significantly:

  8.81%(2001)➔10.74%(2010)*➔37.9%(2050)

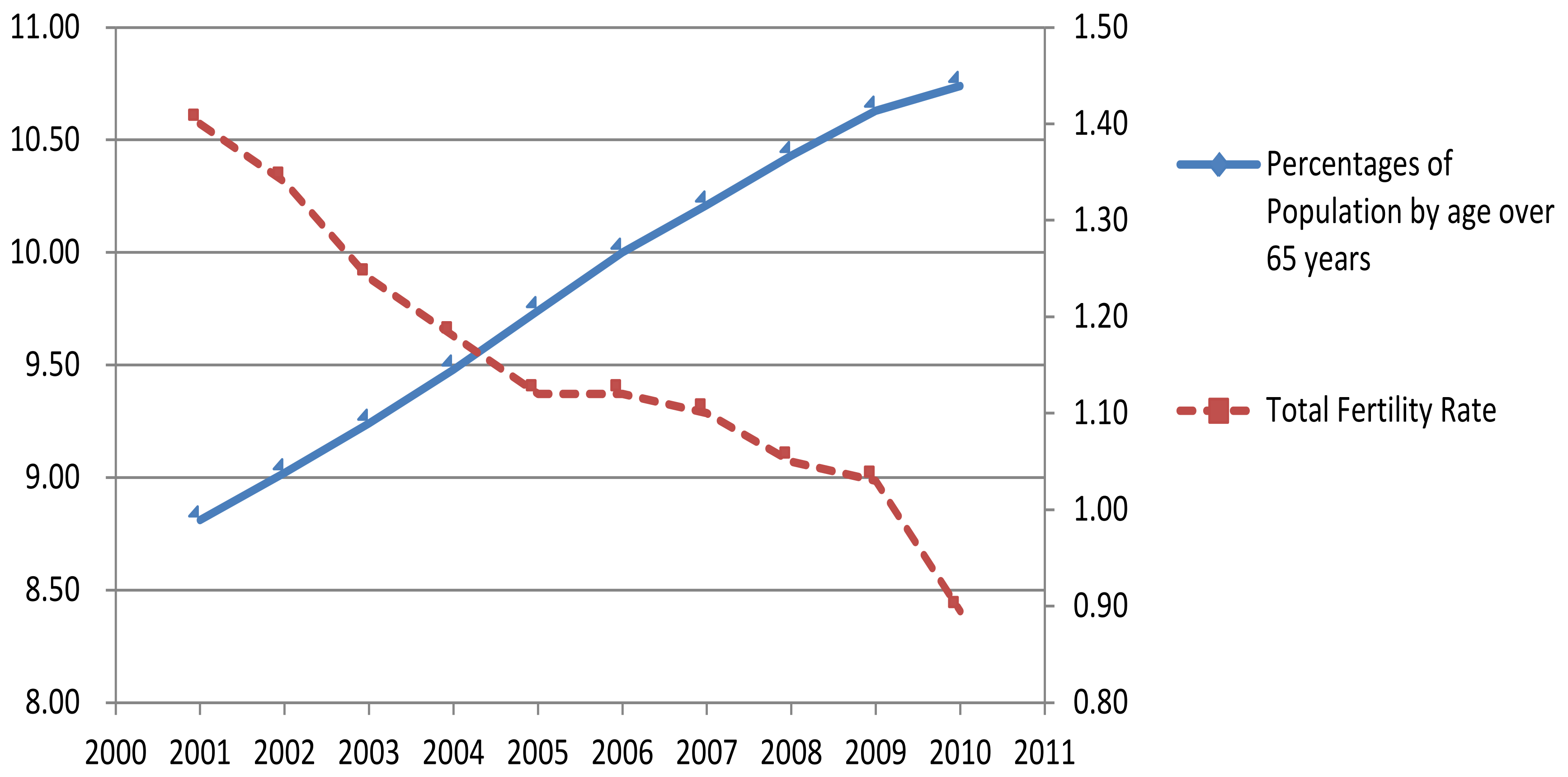
  In the future , the proportion of 65+ is expected to catch up that of Japan within 30 years (more than 30% in 2040).

- Total Fertility Rate:

  1.4(2001)➔0.9(2010)

Motivation

- Percentages of population by age over 65 years and total fertility rate:



Motivation

- What is the problem about the rapid population aging and prolonging life expectancy in Taiwan?
- Workforce reduction? Pensions are inadequate? Health care? Individual Retirement Arrangements?
- How to face the problems of retirement preparation in Taiwan?
- The national pension plan and health insurance enforced.
- Individual ? annuity product.

Motivation

- Taiwan's insurance companies also face the decreasing mortality rate problem.
- The life expectancies in Taiwan are average annual extension of 0.23(male) and 0.27(female) years

--Source: Abridged Life Table In Taiwan 1970-2008

➔A 20-year immediate annuity product ,will lost about 5 years of uncertainty risk.

Motivation

- It would generate financial insolvency for annuity products (i.e., longevity risk).

➔To transfer the uncertainty of mortality risk to capital market probably is the most popular choice.

- However, the accuracy of mortality prediction is crucial to risk transfer and pricing the longevity bond needs to include the uncertainty (or variance) of prediction.

Mortality Models

- The human longevity continues to increase and the mortality rates of same age decrease seemly in a systematic way.--Lee-Carter model (1992) is easy to use and generally has a good fitting accuracy. several modifications have been proposed, for example: Renshaw and Haberman (2006), Cairns et al. (2006a, 2006b), Yang et al. (2009) and Chen and Cox (2009),…

➔Recently, the performance of all mortality models tend to be data dependent.

---The search of mortality models still continues.

---In this study, we propose another alternative for modeling the mortality rates.

Mortality Model and Regular Discount Sequences

- We use the idea of regular discount sequence from the Bandit Problem to construct a mortality model (Wang and Yue, 2011).
- Treating the survival time T as the random variable N in discount sequence $\gamma_n = P(N \geq n)$. That is γ_n as the survival function $S(n) = P(T \geq n)$

Mortality Models

- Why use the discount sequence model?
- Discount sequences in Bandit Problem are used to model the possibility of future rounds.
- Similar to the survival function, the discount sequence is a non-increasing function of time.
- In the same year level , mortality rate has different change tendency in different age.
- The ratio of mortalities (or surviving numbers, …, etc) is a useful descriptive measure.

--for example : (The differences between two age groups are the same) The relative risks of Taiwan male’s mortalities are different.

Mortality Models

- Why use the discount sequence model?

Age Group	1977 year	Age Group	1981 year
55-59	0.07373	100-104	0.90805
60-64	0.11324	105-109	0.94801
Difference	0.040	Difference	0.040
Ratio	1.536	Ratio	1.044

Mortality Model and Regular Discount Sequences

- The relationship between Discount Sequence and mortality is

$$l_n \cdot l_{n+2} \le (l_{n+1})^2 \quad \text{or} \quad \frac{l_n \cdot l_{n+2}}{(l_{n+1})^2} \le 1 \quad ,$$

where l_n is the number of survivors at age n in life table.

- The regularity condition can also be expresses as the form of life expectancy, $e_n \cdot e_{n+2} \le (e_{n+1})^2$

Mortality Model and Regular Discount Sequences

- If the survival time satisfies the regularity condition, the mortality rate is a non-decreasing function of age.
- The idea behind regularity condition is related to log concave function
- The discount sequence approach can cover

 1) Gompertz model

 2) Uniform distribution of death (UDD)assumption

 3) Constant Force (CF) assumption

 4) Hyperbolic assumption

Prediction Methods

- Proposed method for Prediction :
- ◆Cross-Validation: Separating methods into control group and experimental group .
- Experimental Group:

Model I. Suppose we know the true $q_{64}(t)$.

Model II-1. Predicting $q_{64}(t)$ via the LC model.

Model II-2. Predicting $q_{64}(t)$ via the Gompertz model.

Model II-3. Median of 1,000 Block Bootstrap simulation runs for both h_{xt} and r_{xt} , where central mortality rate $m_{xt} = h_{xt} \times r_{xt}$.

Prediction Methods

◆Cross-Validation

- Control Group:

LC model: the logarithm of central mortality rate satisfies

$$\ln(m_{xt}) = \alpha_x + \beta_x \kappa_t + \varepsilon_{xt}$$

Parameters estimate: SVD (singular value decomposition) method.

The parameters α_x and β_x are treated as constants of time and predict the future κ_t:

Model III-1. Combining OLS and AR (1)

Model III-2. OLS without time series trend

Model III-3. Median of 1,000 Block Bootstrap simulation runs

Prediction Methods

- Mortality Projection Comparison:
- Cross-validation: separating the data into training period (or in-sample) and testing period (or out-sample).
- Short-term Forecast Data Description

	Taiwan	Japan
Training Period	1970-2002	1970-2002
Testing Period	2003-2008	2003-2008

- Long-term Forecast Data Description

	England & Wales	Japan	West Germany	U.S.A.
Training Period	1950-1981	1947-1982	1956-1983	1933-1981
Testing Period	1982-2006	1983-2007	1984-2008	1982-2006

Prediction Methods

- Mortality Projection Comparison:

The criteria for forecast accuracy: Mean Absolute Percentage Error (MAPE): $MAPE = \frac{1}{n}\sum_{i=1}^{n}\frac{|\varepsilon_i|}{Y_i}\times 100\%$, where Y_i and ε_i are the observed value and residual of observation i , $i = 1, 2, \ldots, n.$ (Lewis, 1982)

- Short-term Forecast results

1. The LC model has a better predicting accuracy in Japan, than in Taiwan.
2. For the proposed method, knowing the true mortality rate does have a smaller MAPE and can provide very accurate prediction.
3. If we use double BB method, i.e., model II-3, the proposed discount sequence approach is very competitive and is a possible alternative for the LC model.

Prediction Methods

- Short-term Forecast(MAPE)

Model		Taiwan		Japan		Average
		male	female	male	female	
I	1	7.3	4.5	4.2	9.5	6.4
II	1	15.8	6.2	4.8	9.8	9.2
	2	10.5	9.6	8.7	23.4	13.0
	3	8.1	4.6	3.9	8.9	6.4
III	1	15.3	15.4	4.2	16.9	12.9
	2	13.3	10.2	3.3	5.5	8.1
	3	15.1	6.8	3.4	6.8	8.0

Prediction Methods

- Long-term Forecast results

1. using the idea of discount sequence with perfect information (known $q_{64}(t)$) has the smallest MAPE.
2. If $q_{64}(t)$ is unknown, the double BB method give the discount sequence approach the best performance.
3. Either the simple OLS or the simulation based BB method outperforms the AR(1) approach for the LC model.
4. The proposed approach with BB prediction is slightly better than 3 approaches of LC model,

◆consistent with the results in short-term forecast.

Prediction Methods

- Long-term Forecast (MAPE)

Model		England and Wales		Japan		West Germany		U.S.A.		Average
		male	female	male	female	male	female	male	female	
I	1	8.3	5.6	4.7	12.5	15.2	5.1	6.0	12.0	8.7
II	1	22.1	21.1	37.2	61.3	22.1	6.1	19.6	12.1	25.2
	2	36.0	47.4	11.4	13.3	15.7	15.4	11.5	15.6	20.8
	3	14.9	5.8	18.1	27.0	13.4	8.6	7.6	18.9	14.3
III	1	20.7	16.5	33.9	71.7	18.6	15.6	18.4	22.5	27.3
	2	18.0	6.0	16.5	31.0	20.9	10.3	14.0	7.1	15.5
	3	17.2	7.3	20.5	39.9	17.9	4.0	16.0	7.7	16.3
Average		19.6	15.7	20.3	36.7	17.7	9.3	13.3	13.7	18.3

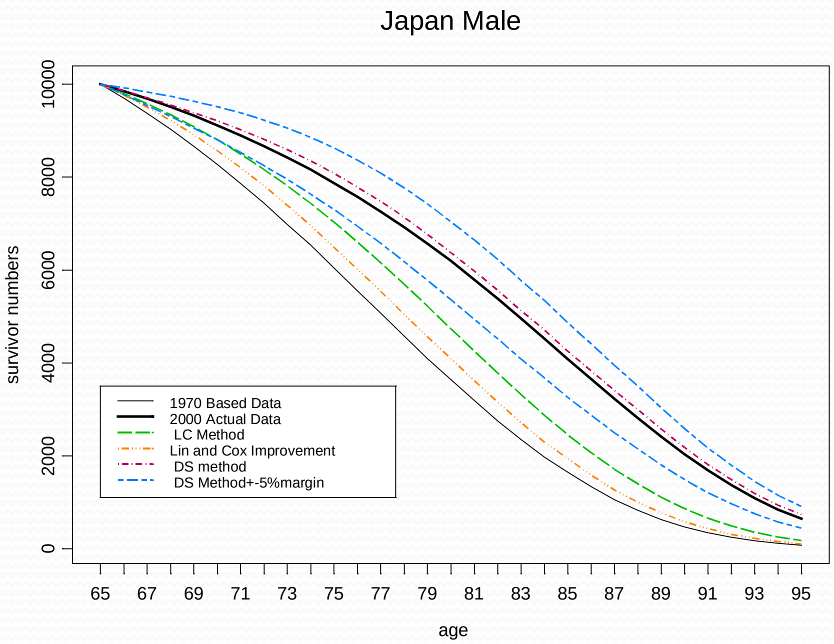
Pricing on longevity bond

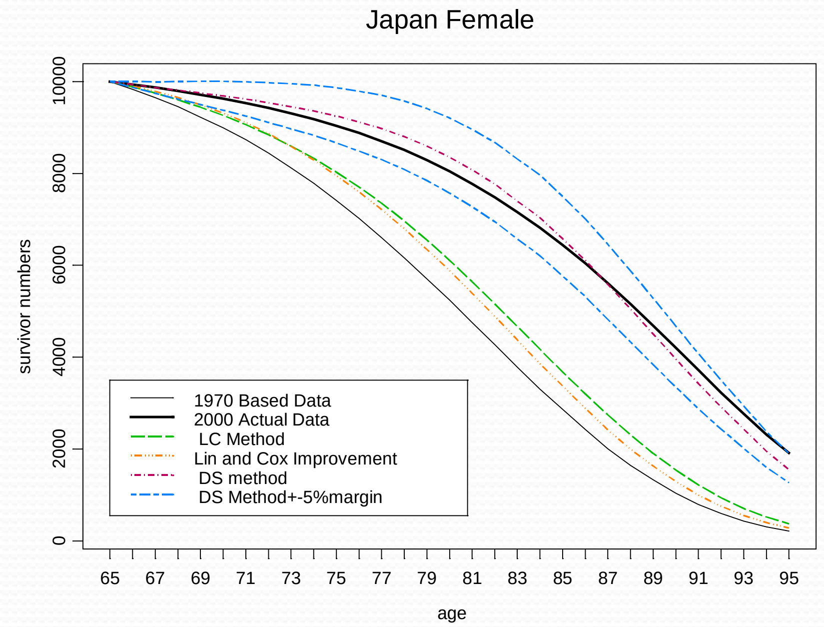
- we use the idea of longevity risk securitization by Lin and Cox (2005) to price the longevity bond.
- The calculation is based on the survivors of ages 65~95 in 1970 from HMD (Human Mortality Database).
- Mortality improvements : Lin and Cox (2005)、LC model、method II-3 (discount sequence; DS)、Actual data
- 10,000 persons
- fixed constant coupon rate 7%

Pricing on longevity bond

- The strike levels comparison

For example: Japan





Pricing on longevity bond

- The strike levels comparison: Japan(Conti.)
- Both the Japanese males and females experienced a big mortality improvement from 1970 to 2000.
- Standard deviation from 1,000 block bootstrap simulation is fairly wide, we shall take the 5% standard deviation as the margins of risk (Risk Tolerance).
- The predicted values of the DS method are very close to the true values in 2000; LC method and Lin & Cox model are relatively conservative and underestimate the true numbers of survivors

Pricing on longevity bond

- Longevity Bond Prices (Japan)
- The bond price will be higher, if we take smaller mortality rates to calculate the strike level. it is more profitable to investors. But, the insurance companies receives less risk-transferring and this is against the purpose of longevity bond.

	Strike levels	Bond price per 1,000
Male	DS–5% margin	969.64
	DS	966.02
	DS+5% margin	918.62
Female	DS–5% margin	967.62
	DS	953.29
	DS+5% margin	931.13

Conclusion and Discussion

- Conclusion
- Common mortality models, such as: Gompertz Law, Uniform Distribution of Death, Constant Force, and Hyperbolic assumption meet the regularity condition.
- The regularity condition is related to log concave function
- Mortality prediction: The proposed discount sequence approach has competitive errors (MAPE) in forecasting mortality rates than the LC model.
- The discount sequence does provide a good choice for modeling mortality rates and pricing the longevity bond.

Conclusion and Discussion

- Conclusion (Conti.)
- The confidence interval of DS method can be the reference of setting strike level. The choice of risk margin depends on the different company's mortality risk tolerance.

- Discussion
- The regularity condition is to normalize mortality rates between ages at the same year, which is a period approach.

---Maybe the discount sequence shall consider introducing the cohort effect.

Conclusion and Discussion

- Discussion (Conti.)
- It shall be possible to find connections between consecutive years of discount sequences and use the relationship to build future values. -- adapt data reduction methods, e.g., the singular value decomposition (Lee and Carter, 1992) or principal component analysis (Yang et al., 2010), and use fewer components (or variables) for forecasting.
- We shall continue exploring the possibility for applying the discount sequence to other longevity bonds
- We shall add interest risk and combine with mortality improvement in pricing.

Thank you for your attention.