Managing Capital Market and Longevity Risks in a Defined Benefit Pension Plan

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Capital Market Risk and Longevity Risk in Defined Benefit Plans

- Capital market risk
  - From the 2008 Coca-Cola annual report:

    "The significant decline in the equity markets and in the valuation of other assets precipitated by the credit crisis and financial system instability has affected the value of our pension plan assets ... We have made and will consider making additional contributions to our U.S. and international pension plans in 2009."

- Longevity risk
  - Companies in the UK understated the aggregate pension liabilities by £40 billion in 2007 due to underestimation of future life expectancy (Cowling and Dales, 2008).

Existing Literature on Pension Risk Management

- Minimizing funding variation (Delong, Gerrard and Haberman, 2008)
- Minimizing solvency risk and contribution rate risk (Haberman and Sung, 1994; Haberman, 1997; Josa-Fombellida and Rincén-Zapatero, 2001; Josa-Fombellida and Rincén-Zapatero (2004)
- Minimizing costs of a fund with a conditional value at risk constraint on underfundings (Bogentoft, Romeijn and Uryasev, 2001)

These papers do not explicitly control total pension cost.

Outline

- We impose a constraint to control the expected total pension cost in the pension asset-liability management.
- Given our model, we analyze how optimal normal contribution and asset allocation will change with different levels of mortality improvement.
- We study how much longevity risk a plan should transfer.
  - Two longevity risk hedging strategies: the ground-up hedging strategy and the excess-risk hedging strategy.

Basic Framework

- Assumptions
  - All members join the plan at the age of x_0 at time 0 and retire at the age of x at time T.
  - The cohort is stable across the entire accumulation phase.
- The value of the accumulated fund at time t, PA_t:

$$PA_t = \sum_{i=1}^{n} A_{i,t-1}(1 + r_{i,t}) \quad \text{for } t \in (0, T]. \tag{1}$$

- The plan's liability at time t, PBO_t:

$$PBO_t = \frac{Ba(x(T))}{(1+\rho)^{T-t}} \quad \text{for } t \in (0, T]. \tag{2}$$

- Denote C as a constant normal contribution (to be determined in optimization problems).

Basic Framework (Cont')

- The pension underfunding/surplus at time t, UL_t:

$$UL_t = PBO_t - PA_t - C \tag{3}$$

- Total pension cost TPC (Maurer, Mitchell and Rogalla, 2009)

$$TPC = \sum_{t=1}^{T} \frac{C + SC_t(1+\psi_1) - W_t(1-\psi_2)}{(1+\rho)^t}, \tag{4}$$

where ρ is the valuation rate. The constants ψ_1 and ψ_2 are penalty factors on supplementary contributions SC_t and withdrawals W_t respectively.

Objective Function and Optimization Problem

$$
\begin{aligned}
\underset{w,C}{\text{Minimize}} \quad & \mathrm{E}\left[(UL_T)^2\right] \\
\text{subject to} \quad & \mathrm{E}\left(UL_T\right) = 0 \\
& \mathrm{E}(TPC) = \zeta \\
& w_i \geq 0, \qquad i = 1, 2, \ldots, n \\
& \sum_{i=1}^{n} w_i = 1 \\
& C \geq 0,
\end{aligned} \tag{5}
$$

where w_i is the weight of pension asset i.

Example

- A cohort joins the plan at age $x_0 = 45$ at $t = 0$.
- They will retire at $T = 20$ at age $x = 65$.
- The initial pension fund $M = \$5$ million at $t = 0$
- Annual retirement benefit of $B = \$10$ million
- The pension funds are invested in three assets:
  - S&P 500 index $A_{1,t}$;
  - Merrill Lynch corporate bond index $A_{2,t}$;
  - 3-month T-bill $A_{3,t}$.
- Pension valuation rate $\rho = 0.08$
- Target expected total pension cost $\zeta = \$24$ million
- Penalty factors on supplementary contributions and withdrawals are both equal to $\psi_1 = \psi_2 = 0.2$

Example (Cont')

- Financial market model
  - The process of each risky asset is described as the combination of a Brownian motion and a compound Poisson process,

$$A_{i,t+\Delta}|\mathcal{F}_t = A_{i,t}\ \exp\left[(\alpha_i - \frac{1}{2}\sigma_i^2 - \lambda_i k_i)\Delta + \sigma_i \Delta W_{it}\right] \prod_{j>N_t^i}^{N_{t+\Delta}^i} Y_{ij},$$

    where $i = 1, 2$.
  - The process of the 3-month T-bill is described as a geometric Brownian motion,

$$A_{3,t+\Delta}|\mathcal{F}_t = A_{3,t}\ \exp\left[(\alpha_3 - \frac{1}{2}\sigma_3^2)\Delta + \sigma_3 \Delta W_{3t}\right].$$

- Data: S&P 500 index, Merrill Lynch corporate bond index and 3-month T-bill monthly data from March 1988 to December 2010.

Example (Cont')

- Stochastic mortality model
  - Lee and Carter (1992) model describes the dynamics of one-year death rate $q_{x,t}$ of age x in year t,

$$\ln q_{x,t} = a_x + b_x \gamma_t + \epsilon_{x,t}$$
$$\gamma_t = \gamma_{t-1} + g + e_t,$$

  where the drift g captures the average mortality improvement rate across all ages per year.
- Data: US male population mortality rates from 1901 to 2007.
- The estimated annual mortality improvement rate $g = -0.20$.

Optimization Results—Base Case

Table: Optimal Pension Normal Contribution and Asset Allocation Strategies with Mortality Improvement Parameter $g = -0.20$ Given $\zeta = \$24$ Million

S&P w_1	Bond w_2	T-Bill w_3	C	$\mathrm{CVaR}_{95\%}$ UL_T	$\mathrm{CVaR}_{95\%}$ TPC
11.13%	44.49%	44.38%	0.67	10.69	28.43

Table: Optimal Pension Normal Contribution and Asset Allocation Strategies with Different Assumptions on Mortality Improvement Parameter g Given $\zeta = \$24$ Million

	S&P	Bond	T-Bill	
g	w_1	w_2	w_3	C
-0.20	11.13%	44.49%	44.38%	0.67
-0.25	12.01%	47.81%	40.18%	0.58
-0.30	12.85%	51.20%	35.95%	0.50
-0.35	13.77%	54.63%	31.60%	0.40
-0.40	14.63%	57.95%	27.41%	0.31

Longevity Risks

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Longevity Risks (Cont')

Table: Optimal Pension Normal Contribution and Asset Allocation Strategies with Different Assumptions on Mortality Improvement Parameter g Given $\zeta = \$24$ Million

g	$\mathrm{E}\left[(UL_T)^2\right]$	$\mathrm{CVaR}_{95\%}(UL_T)$	$\mathrm{CVaR}_{95\%}(TPC)$
-0.20	32.46	10.69	28.43
-0.25	38.00	11.59	28.81
-0.30	44.00	12.47	29.01
-0.35	50.82	13.39	29.36
-0.40	57.95	14.24	29.57

Two Pension Longevity Risk Hedging Strategies

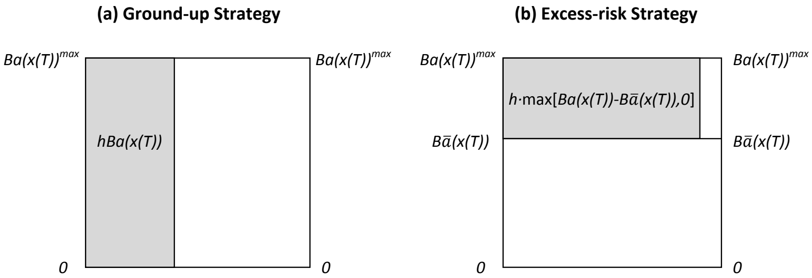

Figure: Two pension longevity risk hedging strategies: the ground-up hedging strategy (on the left) and the excess-risk hedging strategy (on the right)

Ground-Up Hedging Strategy

Table: Optimal Ground-up Hedging Strategies with Different Assumptions on Hedging Cost Parameter δ_1 Given $\zeta = \$24$ Million and $g = -0.20$

δ_1	w_1	w_2	w_3	C	h	$\mathrm{E}\left[(UL'_T)^2\right]$
0	10.59%	42.27%	47.14%	0.690	10.41%	23.720
0.050	11.17%	44.51%	44.32%	0.636	9.91%	26.234
0.150	12.19%	48.43%	39.38%	0.540	9.05%	30.967
0.180	12.11%	48.12%	39.77%	0.553	6.68%	32.289
0.188	11.34%	45.29%	43.37%	0.642	1.51%	32.450
0.190	11.13%	44.49%	44.38%	0.669	0%	32.458

h is the longevity risk hedging ratio.

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Excess-Risk Hedging Strategy with Strike Level $B\bar{a}(x(T))$

Table: Optimal Excess-risk Hedging Strategies with Different Assumptions on Hedging Cost Parameter δ_2 Given $\zeta = \$24$ Million, $g = -0.20$ and the strike level $B\bar{a}(x(T))$

δ_2	w_1	w_2	w_3	C	h	$\mathrm{E}\left[(UL''_T)^2\right]$
0	11.06%	44.25%	44.69%	0.671	100%	31.220
0.050	11.13%	44.50%	44.37%	0.664	100%	31.533
0.150	11.26%	45.01%	43.72%	0.650	100%	32.164
0.190	11.32%	45.22%	43.47%	0.645	100%	32.418
0.195	11.24%	44.94%	43.81%	0.654	60.51%	32.445
0.200	11.16%	44.61%	44.23%	0.665	15.59%	32.457
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Implications for Longevity Securitization

- The market should design attractive longevity securities that can gear up pension plans for those new instruments.
  - The ground-up structure, at least partially, explains the failure of EIB longevity bond in 2004.
  - The excess-risk strategy is more attractive.
    - For example, six longevity swaps were completed in the U.K. in 2009 covering liabilities of approximately £4.1 billion (Brcic and Brisebois, 2010).
- Longevity securities should not be too expensive.
  - The coverage of EIB bond was too expensive (Lin and Cox, 2008)

Conclusion

- This paper proposes a model to identify the optimal contribution, asset allocation and longevity risk hedging strategies that minimize pension funding risk for a DB pension plan.
- We investigate how sensitive the pension funding status is to longevity risk.
  - As the pensioners live longer, the pension plan will make less normal contribute and invest more in risky assets.
- We examine the plan's optimal longevity risk management decision.
  - We compare two longevity risk hedging strategies—the ground-up hedging strategies and the excess-risk hedging strategy.
  - Longevity hedging ratio is negatively related to the hedging cost.