Reinsurance and securitization: Application to life risk management

Pauline Barrieu
Henri Loubergé

Background

- Traditional reinsurance theory (Borch, 1960, 1962):
  - a reinsurance pool is formed to share undiversifiable risk;
  - reinsurers are EU maximizers;
  - they share risk according to their risk-tolerance (Wilson, 1968);
  - risk-sharing is completed by bargaining on side-payments to allocate gains from diversification ($\Rightarrow$ theory of games).
- Over the past 20 years the reinsurance industry was confronted with huge systematic risks…
- Insurance-Linked Securities (ILS) where created to get financial investors involved in the world-wide sharing of risks (see, e.g., cat bonds).

Paper Motivation

- Use the theory of risk measures to provide a general theory of risk-sharing among three representative agents:
  - insurer reinsurer investor
- Apply the theory to a challenging social risk faced by insurers today: longevity risk.
- Investigate the impact of contemplated regulatory measures on risk-sharing.

Longevity risk

- Long-term trend of longevity increases globally, more particularly in developed countries.
- Uncertainty about the future (changes in trends? jumps? or limits to longevity?).
- Already tremendous impact on pensions (pay-as-you-go systems as well as funded systems)
  - ➔ delay of mandatory retirement age.
  - ➔ switch from defined benefit to defined contribution.
- Challenge also for insurers: undiversifiable risk with
  - positive impact on death insurance (if death probability overestimated);
  - negative impact on life insurance (annuities).

Increase in life expectancy in the USA
(Source: Boucher and Boyer, 2009)

Period	1900-1940	1940-1960	1960-1990
Total change	15.9	6.4	5.1
Origin by age			
less than 1 year	30%	22%	24%
1 to 14	28%	11%	6%
15 to 44	23%	25%	6%
45 to 64	6%	17%	27%
65 and more	3%	19%	35%
Multi-generation	11%	6%	2%

Life expectancy of men at 65

(Source: Boucher and Boyer, 2009)

Country	1980	1985	1990	1995	2001	Change
Germany	13.0	14.0	14.7	15.5	16.0	+ 3.0
UK	12.9	14.1	14.8	15.5	16.1	+ 3.2
Australia	13.7	15.2	15.7	16.6	17.2	+ 3.5
Canada	14.5	15.7	16.0	16.5	17.1	+ 2.6
USA	14.1	15.1	15.6	16.1	16.4	+ 2.3
Finland	12.5	13.7	14.5	15.1	15.7	+ 3.2
France	13.6	15.5	16.1	16.5	16.9	+ 3.3
Japan	14.6	16.2	16.5	17.0	17.8	+ 3.2

Life expectancy (2)

- Longevity tables display a systematic trend.
- The trend was underestimated in the past.
- Will it continue at a steady pace or not?
- Moreover, longevity risk has also a specific component: average longevity varies across insured populations.
- This risk is diversifiable using reinsurance.
- The systematic component must however be assumed by the insurance industry or shared with financial investors.

➔ 3-agent model

Three agents

- **Insurer** faces a mortality rate with a systematic and a specific component. Pays a stochastic amount:

$$\hat{X} = X(\Theta) + \overline{X}(\Theta^{\perp})$$

- **Reinsurer** supplies coverage to an amount $\hat{J}$ at price κ and issues a mortality bond with a contingent payoff $M(\Theta)$ at price π.

- **Financial investor** pays π to get the contingent payoff $M(\Theta)$

Decision criterion

- Monetary risk measure $\rho(\Psi)$ with desirable coherence properties (Föllmer and Schied, 2002):
  - positive homogeneity
  - convexity (leading to sub-additivity).
- A risk measure represents a monetary amount which should be put on reserve to compensate for the risk (analogy with *VaR*).
- Modified risk measure $\rho^m(\Psi)$ using a change of probability from $\mathbb{P}$ to $\mathbb{Q}$ to reflect optimal financial investment on the market (Barrieu-El Karoui, 2003).

Assumptions

- Use of entropic (modified) risk measure to perform minimizations:

$$\rho_a^m(\Psi) = \gamma_a \ln E_{\hat{Q}}\left[\exp(-\frac{1}{\gamma_a}\Psi)\right]$$

➔ Link with EU: the entropic risk measure is the opposite of the certainty equivalent of the risk for negative exponential CARA utility: $\rho(x) = - C_x$.

- The reinsurer organizes transfers and minimizes his risk measure under constraints of no increased risk for the insurer and the investor.

Problem

- General formulation:

$$\min_{\hat{J},M} \rho_R^m (W_R - \hat{J} - M(\Theta) + \pi + \kappa)$$

$$\text{s.t. } \rho_I^m (W_I - \hat{X} + \hat{J} - \kappa) \leq \rho_I^m (W_I - \hat{X})$$

$$\rho_B^m (W_B + M(\Theta) - \pi) \leq \rho_B^m (W_B)$$

- Using the cash invariance property of risk measures:

$$\min_{\hat{J},M} \left\{ \rho_R^m (W_R - \hat{J} - M(\Theta)) + \rho_I^m (W_I - \hat{X} + \hat{J}) + \rho_B^m (W_B + M(\Theta)) \right\}$$

Proposition 1 : Optimal contracts

- Optimal risk-sharing of specific risk:

$$\bar{J}^*(\Theta^\perp) = \frac{\gamma_R}{\gamma_R + \gamma_I}\bar{X}(\Theta^\perp) + \text{ constant}$$

- Optimal risk-sharing of systematic risk:

$$J^*(\Theta) = \frac{\gamma_R + \gamma_B}{\gamma_I + \gamma_R + \gamma_B}X(\Theta) + \text{ constant}$$

$$M^*(\Theta) = -\frac{\gamma_B}{\gamma_I + \gamma_R + \gamma_B}X(\Theta) + \text{ constant}$$

Transaction feasibility

- Let π_B represent the maximum buying price for the bond.
- Let π_R represent the minimum selling price of the reinsurer for the bond.
- *Proposition 2*: $\pi_B - \pi_R > 0$.

Regulatory constraints

- Threat of increased regulatory constraints on insurance/reinsurance activity in the aftermath of the subprime crisis and AIG failure.
  - ➔ More stringent reserve requirements (inflated risk measures), with Solvency II.
  - ➔ Possible penalty on securitization activity for reserve requirements purpose.
  - ➔ Possible minimum risk-retention requirements for insurers and reinsurers issuing ILS.
- Two possible regulations considered in the paper.

Case 1: Different reserve requirements on securitization

- The minimization program is now:

$$\min_{\hat{J},M} \rho_R^m(W_R - \hat{J} + \kappa) + \alpha\rho_R^m(-M(\Theta) + \pi)$$

$$\text{s.t. } \rho_I^m(W_I - \hat{X} + \hat{J} - \kappa) \leq \rho_I^m(W_I - \hat{X})$$

$$\rho_B^m(W_B + M(\Theta) - \pi) \leq \rho_B^m(W_B)$$

- Due to separation between insurance risk-sharing and securitization, the optimal contracts are now:

$$\hat{J}^* = \frac{\gamma_R}{\gamma_R + \gamma_I}\hat{X} \qquad M^*(\Theta) = 0$$

- The rationale for securitization disappears and reinsurance capacity is reduced.

Case 2 : Penalty on securitization

- ILS are now inflated (or deflated) by a factor α for risk management purposes.
- The minimization problem is now:

$$\min_{\hat{J},M} \rho_R^m (W_R - \hat{J} + \kappa + \alpha[-M(\Theta) + \pi])$$

$$\text{s.t. } \rho_I^m (W_I - \hat{X} + \hat{J} - \kappa) \leq \rho_I^m (W_I - \hat{X})$$

$$\rho_B^m (W_B + M(\Theta) - \pi) \leq \rho_B^m (W_B)$$

- With results: $\bar{J}^*(\Theta^\perp) = \dfrac{\gamma_R}{\gamma_R + \gamma_I} \bar{X}(\Theta^\perp) +$ constant

Case 2 (cont’d)

$$J^*(\Theta) = \frac{\gamma_R(\gamma_R + \alpha\gamma_B)}{\gamma_R(\gamma_R + \alpha\gamma_B) + \gamma_I(\gamma_R + (\alpha - 1)\gamma_B)} X(\Theta) + \text{ constant}$$

$$M^*(\Theta) = -\frac{\gamma_B\gamma_R}{\gamma_R(\gamma_R + \alpha\gamma_B) + \gamma_I(\gamma_R + (\alpha - 1)\gamma_B)} X(\Theta) + \text{ constant}$$

- *Proposition 6*: J^* and M^* are increasing in α. In this case, imposing $\alpha > 1$ ($\alpha < 1$) on securitization reporting will increase (decrease) securitization and reinsurance transfers.

Conclusions

Two main conclusions.

1. Risk minimization leads to dual risk management:
   - traditional risk-sharing of specific risk between insurers and reinsurers;
   - risk-sharing of systematic risk between insurers, reinsurers and financial investors.
2. Strong sensitivity of systematic risk transfer to regulation on securitization activity. Depending on the kind of restrictions imposed on ILS underwriting, reinsurance capacity may be boosted, reduced or brought to zero.