Managing Systematic Mortality Risk with Group Self Pooling and Annuitisation Schemes

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Introduction

- Lack of well developed life annuity market in Australia and recommendations for longevity products. (Henry Tax Review, Cooper Review)
- Large pools of savings in Industry funds without longevity products
- Concerns with life annuity markets – systematic longevity risk, capital costs and loadings, insurer solvency risk
- Mutual risk pooling solutions such as Group Self-annuitisation schemes (GSA) an alternative – lower costs, mutual risk sharing of mortality/longevity risk.
Research Objectives

- Assess GSA longevity risk pooling
  - Systematic mortality risk and how to manage in a GSA pool
  - Size of pool and impact on annuity payments
  - Sharing risk fairly across new cohorts
  - Managing older age variability from survival distribution
  - Taking into account expected future systematic mortality improvements
**Literature**

- Annuitisation is optimal for individual longevity risk management. (Yaari 1965; Davidoff et al. 2005)
- GSAs
  - Pooling longevity risk (Piggott, Valdez and Detzel 2005)
  - Adverse selection (Valdez, Piggott and Wang 2006)
  - Portfolio choice and welfare gains (Stamos 2008)
- Research issues
  - Effective management of systematic risk borne by members
  - Payment amount declines and volatility rises over time, especially at extreme ages
  - Reducing numbers in pools, in old ages, increases payment volatility
Summary of key results

- Simple GSA pooling mechanism produces declining payments and high variability over time, due to mortality dependence and systematic mortality improvements.

- Solutions that are assessed:
  - Pool size effect on idiosyncratic risk and the “tontine” or “lucky hump” volatility
  - Old age variability with the introduction of younger cohorts at the same age, rather than at the same time, pooling mortality experience
  - Explicitly allowance for expected future systematic mortality improvements in annuity payments rather than sharing them as they are realised through time
Annuity Payment in GSA

- GSA benefit determination:
  - On contribution, the initial fund balance of the member is
    \[ \sum_{k=0}^{x} F_{i,t} \]
    where \( x \) is the age for the \( i \)-th individual at time \( t \), being a member of the pool for \( k \) years
  - The member's benefit payment at any given point in time is
    \[ k_x B_{i,t}^\ast = \frac{k \hat{F}_{i,t}}{\hat{a}_{x+k,t+k}} \]
GSA Benefit Determination

- Each year, the member's fund is accumulated with interest and investment earnings.

\[
k^x F_{i,t} = \left( \frac{1}{x} \hat{F}_{i,t-1}^x - \frac{1}{x} B_{i,t-1}^x \right) e^{\delta}
\]

- For living members, the accumulated fund balance is supplemented by the fund balance of members who die

\[
k^x \hat{F}_{i,t}^* = \frac{k^x F_{i,t}^*}{p_{x+k-1,t-1} \sum_{x} \sum_{A_t} \frac{k^x F_{i,t}^*}{p_{x+k-1,t-1}}} \cdot F_t^*
\]

\(\delta\) is the interest rate or investment returns, \(A_t\) is the number of living members at time \(t\).
GSA Benefit Computation

More general computation of benefit payments in successive periods.
Determined by an adjustment factor reflecting the mortality experience of the pool, as well as systematic improvements that have taken place.

\[
k B_{i,t}^* = (k-1) B_{i,t-1}^* \times \frac{F_t^*}{\sum_{k \geq 1} \sum_x \frac{1}{p_{x+k-1,t-1}} \frac{\hat{a}_{x+k,t}}{a_{x+k,t-1}} \sum A_t k F_{i,t}^*}
\]

Compared to Piggott et al.

\[
k B_{i,t}^* = (k-1) B_{i,t-1}^* \times \frac{F_t^*}{\sum_{k \geq 1} \sum_x \frac{1}{p_{x+k-1}} \sum k F_{i,t}^*}
\]
GSA Benefit Computation – Allowing for Improvements

Future expected mortality improvements are taken into account.

\[\ddot{a}_{x,t} = \sum_{k=0}^{\infty} e^{-\delta k} E[k p_{x,t}]\]

\[E[k p_{x,t}] = \prod_{s=0}^{k-1} E[p_{x+s,t+s} \mid \mu_{x,t}]\]

Rather than waiting for them to be realised

\[\ddot{a}_x = \sum_{k=0}^{\infty} e^{-\delta k} k p_x\]
Systematic Mortality Model

- Stochastic 2-factors GoMa model based on Schrager 2006
  \[ \mu_{x,t} = Y_{t1} + Y_{t2}c^x \]
  \[ dY_{t1} = a_1 dt + \sigma_1 dW_{t1} \]
  \[ dY_{t2} = a_2 dt + \sigma_2 dW_{t2} \]
  \[ dW_{t1} dW_{t2} = \rho dt \]
  with the initial condition \[ \mu_{x,0} = y_1 + y_2 c^x \]

- Key model features:
  - Systematic dependence through time dynamic factors
  - Common improvements through the \( c \) factor
  - Extreme value distribution for survival times
  - Closed form annuity factors (or simple approximations)
  - Negative mortality rates - removed in simulations resulting in autoregression
Pool Size and the Tontine or “Lucky Hump”

Simulation results for 10 Australian males at 5% interest, $100 initial contribution each. The grey bands indicate 95% simulation errors from 5000 simulations.
Pool Size and the Tontine or “Lucky Hump”

1000 members. The “lucky hump” disappears to the right, and payments decline less. Volatility in old ages remains an issue.
Dynamic Pooling with multiple cohorts of same entry age

Introduce younger cohorts regularly, pooling mortality experience. The “lucky hump” disappears, payment amounts decline less and variability at old ages is reduced.
Take into account expected mortality improvements

Taking into account expected future systematic improvements. The median payments no longer decline.
Inflation indexed payments and real Interest rates

Assuming real interest rate of 2.69% (20 year historical average), inflation adjusted benefit payments increase over time.
Uncertainty from real interest rate variability

Variability in the real interest rate using a Cox-Ingersoll-Ross model. Inflation adjusted benefit payments display increased variability.
Conclusions and Summary

- Size of pool and idiosyncratic risk: increasing the pool size increases the amount and decreases the volatility of benefit payments received by GSA members (reduces Tontine effect).
- Old age volatility: introducing new cohorts from younger cohorts on a regular basis reduces the volatility of benefit payments received by older members.
- Systematic risk: Taking into account future mortality improvements results in more stable benefit payment amounts for GSA members.
- Investment variability increases the volatility of benefit payments received by GSA members; but investment returns can be shared in a GSA, not in an ordinary annuity.
- Systematic and older age mortality risks remain. Other solutions such as reinsurance, investment in longevity bond to manage systematic risk or hedging systematic longevity risk using financial markets with securitisation.
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