A Two-Population Mortality Model with Transitory Jump Effects

(Joint work with Johnny S.H. Li and Ken Seng Tan)

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Outline

Motivation

Mortality jumps

- E.g. Spanish flu epidemic in 1918
- Important for mortality modeling and forecasting
- Important for pricing mortality-linked securities, especially those for hedging extreme mortality risk
- Modeling mortality jumps: Biffis (2005), Lin and Cox (2005), Chen and Cox (2009) and Cox et al. (2010)

Motivation (cont'd)

Multi-population models

- Model the potential correlations across different populations
- Ensure biologically reasonable mortality forecast
- Allow evaluating population basis risk
- Two-population mortality models: Carter and Lee (1992), Li and Lee (2005), Li and Hardy (2011), Dowd et al. (2011) and Cairns et al. (2011)

Motivation (cont'd)

Our Work

- Introduce jump effects to a two-population mortality model with a Lee-Carter structure
- Investigate the impact of mortality jumps on the securitization of mortality risk

A Two-Population Model without Jump Effects

$$\ln(m_{x,t}^{(i)}) = \alpha_x^{(i)} + \beta_x^{(i)} \kappa_t^{(i)} \quad i = 1, 2$$

- $m_{x,t}^{(i)}$: central death rate for population i at age x and in year t
- $\kappa_t^{(i)}$: period effect index for population i in year t
- $\alpha_x^{(i)}$: average level of mortality for population i at age x
- $\beta_x^{(i)}$: sensitivity to $\kappa_t^{(i)}$ for population i at age x

Non-divergence

Necessary conditions

1. $\beta_x^{(1)} = \beta_x^{(2)}$ for all x
2. $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$ do not diverge over the long run

Non-divergence (cont'd)

Modeling $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$

$$\begin{aligned} \kappa_{t+1}^{(1)} &= \kappa_t^{(1)} + \mu_\kappa + Z_\kappa(t+1) \\ \Delta_\kappa(t) &= \kappa_t^{(1)} - \kappa_t^{(2)} \\ \Delta_\kappa(t+1) &= \mu_{\Delta_\kappa} + \phi_{\Delta_\kappa}\Delta_\kappa(t) + Z_{\Delta_\kappa}(t+1) \end{aligned}$$

- $|\phi_{\Delta_\kappa}| < 1$
- $(Z_\kappa(t), Z_{\Delta_\kappa}(t))' \sim \text{BVNorm}((0,0)', V_Z)$

Model fitting

A two-stage approach

1. Estimate parameters $\alpha_x^{(1)}$, $\alpha_x^{(2)}$, β_x, $\kappa_t^{(1)}$ and $\kappa_t^{(2)}$
2. Estimate the parameters in the time-series processes for $\kappa_t^{(1)}$ and $\Delta_\kappa(t)$

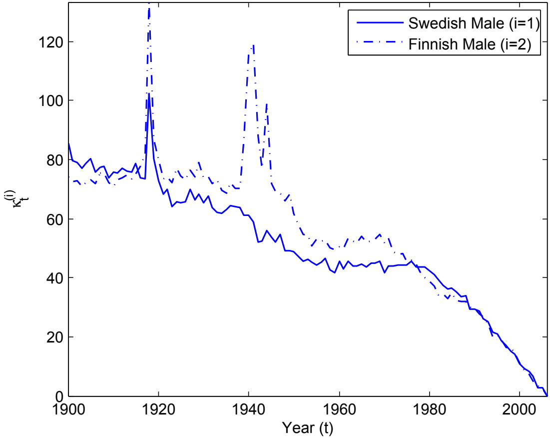


Figure: Estimates of the period effect indexes for Swedish male and Finnish male populations. (Data: sample period of 1900 to 2006 and sample age range of 25 to 84, obtained from Human Mortality Database (2011))

Modeling nonconcurrent transitory jumps

$$\begin{aligned}
\kappa_{t+1}^{(1)} &= \hat{\kappa}_{t+1}^{(1)} + N_{t+1}^{(1)} Y_{t+1}^{(1)} \\
\kappa_{t+1}^{(2)} &= \hat{\kappa}_{t+1}^{(2)} + N_{t+1}^{(2)} Y_{t+1}^{(2)} \\
\hat{\kappa}_{t+1}^{(1)} &= \hat{\kappa}_{t}^{(1)} + \mu_{\kappa} + Z_{\kappa}(t+1) \\
\hat{\Delta}_{\kappa}(t) &= \hat{\kappa}_{t}^{(1)} - \hat{\kappa}_{t}^{(2)} \\
\hat{\Delta}_{\kappa}(t+1) &= \mu_{\Delta_{\kappa}} + \phi_{\Delta_{\kappa}} \hat{\Delta}_{\kappa}(t) + Z_{\Delta_{\kappa}}(t+1)
\end{aligned}$$

- $\{\hat{\kappa}_t^{(i)}\}$: unobserved period effect index that is free of jumps
- $(Y_t^{(1)}, Y_t^{(2)})'$: jump severities $\sim$ BVNorm$((\mu_Y^{(1)}, \mu_Y^{(2)}), V_Y)$
- $(Z_{\kappa}(t), Z_{\Delta_{\kappa}}(t))'$: error terms $\sim$ BVNorm$((0,0), V_Z)$

Modeling nonconcurrent transitory jumps (cont'd)

- $N_t^{(i)}$: jump count for population i
  - $P(N_t^{(1)} = 1, N_t^{(2)} = 1) = p_1$
  - $P(N_t^{(1)} = 1, N_t^{(2)} = 0) = p_2$
  - $P(N_t^{(1)} = 0, N_t^{(2)} = 1) = p_3$
  - $P(N_t^{(1)} = 0, N_t^{(2)} = 0) = 1 - p_1 - p_2 - p_3$

Parameter estimates

Parameters	Nonconcurrent-jump	No-jump
μ_κ	-0.6372	-0.8057
μ_{Δ_κ}	-0.1269	-0.9803
ϕ_{Δ_κ}	0.9184	0.8486
$V_Z(1,1)$	4.1752	17.3162
$V_Z(1,2)$	1.0633	-11.0413
$V_Z(2,2)$	2.7786	38.1830
$\mu_Y^{(1)}$	3.5824	N/A
$\mu_Y^{(2)}$	12.4430	N/A
$V_Y(1,1)$	116.3952	N/A
$V_Y(1,2)$	184.9353	N/A
$V_Y(2,2)$	293.8356	N/A
p_1	0.0622	N/A
p_2	0	N/A
p_3	0.0496	N/A

Likelihood ratio test

- Null model: no-jump model (log-likelihood $l_1 = -634.1980$)
- Alternative model: nonconcurrent-jump model (log-likelihood $l_2 = -503.0726$)
- Test statistics: $2(l_2 - l_1) = 262.2508$
- Degree of freedom: 8
- P-value: 0

An illustrative trade

- Agent A has life insurance liability $L_t = 5\sum_{x=25}^{44} q_{x,t}^{(2)}$, where $q_{x,t}^{(2)} = 1 - e^{-m_{x,t}^{(2)}}$
- Agent A issues mortality bond
  - 3 year maturity
  - a coupon at the end of each year at a rate of 4.5%
  - principal repayment linked to the index $I_t = \frac{1}{20}\sum_{x=25}^{44} m_{x,t}^{(1)}$
  - principal repayment $= \max\left(1 - \sum_{t=2007}^{2009} \text{loss}_t, 0\right)$
  - $\text{loss}_t = \frac{\max(I_t - 1.3I_{2006}, 0) - \max(I_t - 1.4I_{2006}, 0)}{0.1I_{2006}}$
- Agent B invests in mortality bond

Pricing results

- Price the mortality bond by the economic pricing framework proposed by Zhou, Li and Tan (2010)

Model	Price	Quantity
No-jump	1.0178	0.2249
Nonconcurrent-jump	1.0087	0.1699

Determinants of Supply and Demand

- v_L: the accumulated values of the insurance liabilities
- v_H: the accumulated values of the payouts from one unit of the bond

Determinants

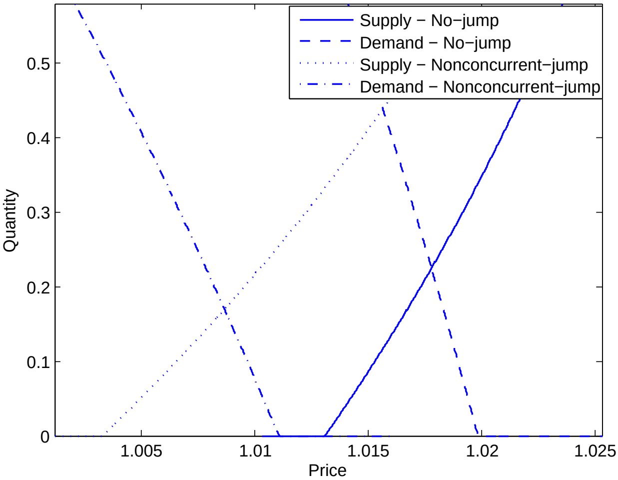
- μ_H: the expected value of v_H
- σ_H: the volatility of v_H
- σ_L: the volatility of v_L
- ρ: the correlation between v_L and v_H

Estimates of μ_H, σ_H, σ_L and ρ

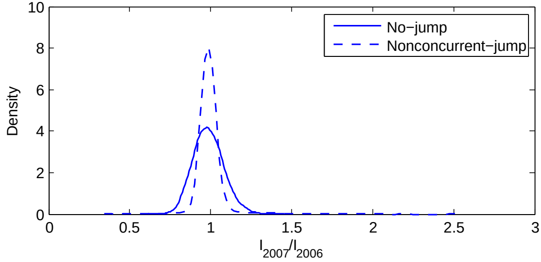
Model	μ_H	σ_H	σ_L	ρ
No-jump	1.0198	0.1258	0.1102	-0.3829
Nonconcurrent-jump	1.0110	0.1573	0.0677	-0.5832

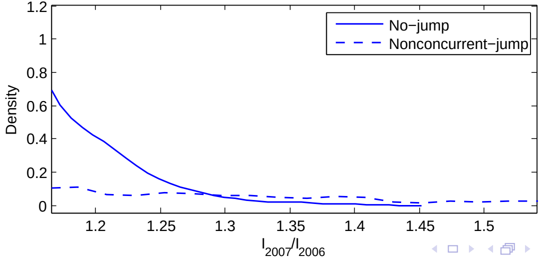
	$\mu_H \downarrow$	$\sigma_H \uparrow$	$\sigma_L \uparrow$	$\lvert\rho\rvert \uparrow$
Supply	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$
Demand	$\downarrow$	$\downarrow$	—	—

The Supply and Demand Curves

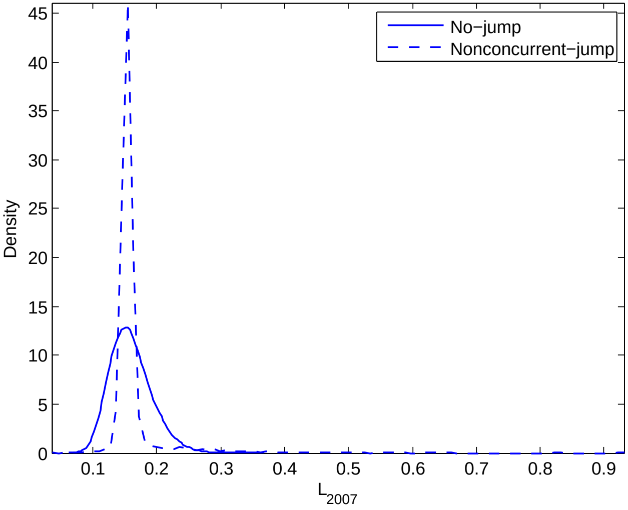


Kernel smoothed density functions for l_{2007}/l_{2006}





Kernel smoothed density functions for L_{2007}



Conclusion

- Incorporating nonconcurrent transitory jumps significantly improves the fit
- It also has significant impacts on the estimated mortality bond price

Future work

- Measure population basis risk
- Allow permanent jumps

THANKS!