Gaussian Forward Mortality Factor Models: Specification, Calibration, and Application

Daniel Bauer*
Department of Risk Management and Insurance, Georgia State University
35 Broad Street, 11th Floor; Atlanta, GA 30303; USA
Email: dbauer@gsu.edu

Nan Zhu
Department of Risk Management and Insurance, Georgia State University
35 Broad Street, 11th Floor; Atlanta, GA 30303; USA
Email: nzhu1@student.gsu.edu

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Extended Abstract

Two of the most important challenges for the application of stochastic mortality models in life insurance practice are (1) the apparent incompatibility of most stochastic methods with classical life contingencies theory, which presents the backbone of insurers' EDP systems, and (2) the complexity of many of the proposed approaches. These obstacles have not only led to an increasing discrepancy between life insurance research and actuarial practice and education, but the reluctance of practitioners to rely on stochastic mortality models may also be a primary reason for the sluggish development of the longevity-linked capital market. In particular, stochastic methods are necessary to assess a company's capital relief when hedging part of their longevity risk exposure, which should be one of the key drivers for the demand of longevity-linked securities.

One model class that may overcome these problems are so-called *forward mortality models*, which infer dynamics on the entire age/term-structure of mortality. As already pointed out by Milevsky and Promislow (2001), the "traditional rates used by actuaries" really are *forward rates* so that such an approach can be viewed as the natural extension of traditional actuarial theory. In particular, the actuarial present values for basic insurance products such as term-life insurance, endowment insurance, or life annuity contracts are of the same form as in classical actuarial theory, where the survival probabilities now are to be interpreted as expected values of realized survival probabilities (cf. Bauer (2009)). Hence, the inclusion of such models in the operations of a life insurer or a pension fund will not require alterations in the management of traditional product lines, but nonetheless present a coherent way to take mortality risk into account where necessary. Examples include the pricing and risk-management of products with mortality-linked guarantees and the calculation of economic capital based on internal models.

*Corresponding author. Phone: +1 (404)-413-7490. Fax: +1 (404)-413-7499.

However, only very few forward mortality models have been proposed so far, and most authors have relied on "qualitative" insights and/or modeling convenience for determining suitable specifications. Moreover, the presented models entail a high degree of complexity because they simultaneously capture the evolution of the entire age/term-structure – or forward surface – of mortality, which leads to several problems in their calibration (see Bauer et al. (2008)).

The current paper overcomes these shortcomings by relying on Gaussian forward mortality factor models, the (necessary) explicit functional form of which has been identified by Bauer et al. (2010). More specifically, similarly to Angelini and Herzel (2005) for interest rate models, we use available data on the term structure of mortality rates such as generational life tables and available annuity price data for the selection of the number of factors and the volatility functions using principal component analysis. These specifications can be readily calibrated to abundant period mortality data based on recursive Bayesian estimation in the form of a Kalman filter. We apply the resulting models to the valuation and analysis of guaranteed annuity options and to the calculation of the solvency capital requirement for a stylized life insurance company.

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