Securitization and Tranching Longevity and House Price Risk for Reverse Mortgages

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Abstract

Reverse mortgages are new financial products that allow the elders to convert their home equity into cash until they die. From the provider's perspective, longevity risk and house price risk are the major risks involved with reverse mortgages. This paper proposes a securitization method to transfer the risks associated with reverse mortgages and focuses on tranching longevity and house price risks for different investors. The structure of securitization for reverse mortgages is similar to that for collateralized debt obligation(CDO). Different to price CDO, we model the dynamics of future mortality and house price instead of default rate. Thus, we model the house price index using ARMA-GARCH process. To deal with longevity risk for elders, we use the CBD model (Cairns et al, 2006) to project future mortality. We propose a risk neutral valuation framework and employ the conditional Esscher transform to price the fair spreads for different tranche investors. The problems of using static mortality table and model risk on pricing fair spread are investigated numerically.

Keywords: Reverse mortgage; GARCH Process; Esscher transform

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1.Introduction

Human longevity has been increasing significantly since the start of the 20th century. Whether human longevity will continue to improve in the future is debating. The view that longevity will continue to increase is supported by the mortality experience in many developed and developing countries (Tuljapurkar et al., 2000; Blake et al., 2008; Yang et al., 2009). Thus, how to increase the retirement income to maintain the elder's living standard has become an important issue. The pension system was the main financial resource for the elders to spend for life. Due to the phenomenon of ageing population and increases in longevity, the pension and annuity providers are suffered substantial financial problem and start to reduce the pension benefit in response (Antolin, 2007; Bauer and Weber, 2007). Governments are now facing a great challenge for financing an ageing population. Therefore, the development of innovative financial products in private market to increase retirement income is needed.

Home equity has been found to be a major asset of individuals in many countries at retirement. For example, in Australia[2], total owner-occupied home equity was AUD$887 billion with those over the age of 60, accounting for AUD$345 billion (39%) of this amount; in the US, the American Housing Survey[3] shows that more than 12.5 million elderly have no mortgage debt, and the median value of mortgaged-free homes is US $127,959. Reverse mortgage is a new financial product that allows the elders to convert a proportion of the equity in their home into cash until they die. Kutty(1998) indicates that the use of home equity conversion mortgage

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[2] Senior Australians Equity Release Association of Lenders Industry Submission, 2005.

[3] American Housing Survey for the United States (2005), Current Housing Reports, H150/05. US Department of Housing and Urban Development and US Census Bureau, Aug. 2006, P156.

products could possibly raise about 29% of the poor elderly homeowners in the U.S. above the poverty line. In the United States, the department of Housing and Urban Development (HUD) first introduced the Home Equity Conversion Mortgage (HECM) program in 1989. In addition to the U.S. market, reverse mortgage products are also found in the U.K., Australia and in Asian countries of Singapore and Japan.

Reverse mortgages differ from traditional mortgages in the way that the loans and accrued interests are repaid once when the borrower dies or leave the house. Unlike traditional mortgage pools, the credit risk in reverse mortgage pools is not driven by potential default of the loans. The main risk factors are the mortality, interest rate and the value of the underlying property. If a borrower lives longer than expected or the decrease in house price, the principal advances and interest accruals may drive the loan balance above the proceeds of sale the property. Thus, reverse mortgage is considered to be longevity dependent asset. To develop reverse mortgage products, risk management has become important for the RM provider to control and manage the risk. Traditional methods for dealing with the risks associated with reverse mortgages are using insurance or writing no-negative guarantee. For example, the HECM program in the United States. The borrower pays a front fee of 2% of the initial property value as the initial property value. The lenders under this program are protected against the losses arising when the loan balance exceeds the equity value at time of settlement. In Britain, most of roll-up mortgages are sold with a no-negative-equity-guarantee (NNEG) that protects the borrower by capping the redemption amount of the mortgage at the lesser of the face amount of the loan and the sale proceeds of the home. The NNEG can be viewed as a European put option on the mortgaged property. Li et al.(2009) develop a framework for pricing and managing the risks for the NNEG.

Securitization is a new financial innovation to hedge longevity risk. Blake and Burrows (2001) were the first to advocate the use of mortality-linked securities to transfer longevity risk to capital markets. The EIB/BNP longevity bond was the first securitization instrument designed to transfer longevity risk but was not issued finally and remained theoretical. Survivor swaps, survivor futures and survivor options have been studied by both academics and practitioners (Blake et al., 2006; Dowd et al., 2006, Biffis and Blake, 2009; Blake et al. 2010). The first derivative transaction, a *q*-forward contract, was issued in January 2008 between Lucida[4] and JPMorgan (Coughlan et al., 2007); the first survivor swap executed in the capital markets between Canada life and a group of ILS and other investors in July 2008. In this context, the valuation of mortality-linked securities represents an important research topic for the development of capital market solutions for longevity risk.

Securitization of longevity risk for annuity business and pension plans have been widely discussed (Macminn et al., 2006; Michael and Wills, 2007). Securitization of reverse mortgage is still in the early developing stage (Zhai, 2000). Wang et al.(2007) illustrate a securitization method to hedge the longevity risk inherent in reverse mortgage products. They study both of survivor bonds and a survivor swap and demonstrate that securitization can provide an efficient and economical way to hedge the longevity risk in reverse mortgages. Sherris and Wills(2007) point out that structuring of longevity risk through a special purpose vehicle requires consideration of how best to tranche the risk in order to meet different market demands. The existing literature illustrates the structure and pricing longevity bond for annuity business(Liao et al. 2007; Wills and Sherris, 2010) not for reverse mortgages. This article attempts to fill this gap. We further consider the tranche design of longevity

[4] A UK-based pension buyout insurer.

security for a portfolio of reverse mortgages and illustrate the pricing of fair spreads for different tranche investors. The structure of the securitization for longevity risk is similar to the collateralized debt obligation (CDO) for credit risk. Thus, we call it as a collateralized reverse mortgage obligation (CRMO) in this research. Different to the survivor swap, the design of CRMO consists on tranching and selling the risk of the underlying portfolio of reverse mortgages. The lender of reverse mortgages, investment bank or insurance company, decides to buy protection against the possible losses due to the longevity risk of the underlying borrower (homeowner). The special purpose company designs the security with the payoff depending on the uncertainty of future losses on the underlying reverse mortgage and tranche the risks to different investors.

To price the fair spread of CRMO for different tranches, we assume a pool of reverse mortgages. Among the reverse mortgage product in the US market, the home equity conversion mortgage(HECM) program is considered the most popular one, which accounts for 95% of the market (Ma and Deng, 2006). Thus, we assume the pools of loans are under the HECM program. To model the loss for HECM program, we need to consider both longevity risk and house price risk. The CBD model (Cairns et al., 2006) is used to capture the dynamic of future mortality for borrowers. To capture the properties of autocorrelation and volatility clustering that are found in the literature(Crawford and Fratantoni, 2003; Miller and Peng, 2006; Chen et al., 2010), we employ ARMA-GARCH process to model the house price dynamic. The risk neutral pricing framework for the CRMO is derived using conditional Esscher transform. Since mortality modeling plays an important role in pricing longevity securities, we also study the impact of mortality modeling on pricing the fair spread for CRMO. We consider the Lee-carter model(1992) to calculate the result. In addition, the earlier HECM program uses static mortality tables to calculate the loan value. We also investigate the effect of failing to capture the dynamics of mortality on securitization of longevity risk for reverse mortgages,

The structure of this paper is organized as follows:

2. Modeling the Risks for HECM Program

2.1 HECM Program

In the US market, HECM program, Fannie Mae's Home Keeper program, and Financial Freedom's Cash Account Advantage are three major reverse mortgage programs. HECM Program was authorized by Department of Housing and Urban Development (HUD) in the Housing and Community Development Act of 1987. Because the HECM program is insured by the US federal government, it is the most popular reverse mortgage program and accounts for 95% of the market (Ma and Deng, 2006).

Under the HECM program, the borrower must be at least 62 years age, living in a single family property that meets HUD’s minimum property standard. The loan can be taken as four common repayment forms including lump-sum, line of credit, tenure, and term. The initial loan amount that can be borrowed depends on the initial loan principal limit (IPL), which is decided according to the borrower’s age, property value and interest rate.

From the lender’s perspective, the loss occurs when the borrower lives longer than expected or the decrease in house price, the principal advances and interest accruals may drive the loan balance above the proceeds of sale the property. To protect the lenders from possible losses, HUD provides mortgage insurance for the HECM program. The mortgage insurance premiums are paid by borrowers and include an upfront premium of 2% of the adjusted property value and an annual rate of 0.5% of the loan outstanding balance as long as the loan is active. Different to mortgage insurance, in this research, we propose a securitization method to hedge longevity risk for reverse mortgage products. Therefore, we illustrate using the HECM program.

2.2 Major Risks in HECM Program

Reverse mortgages differ from traditional mortgages in the way that the loans and accrued interests are repaid once when the borrower dies or leave the house. Thus, unlike traditional mortgage pools, the credit risk in reverse mortgage pools is not driven by potential default of the loans. Longevity risk, interest rate risk and house price risk are the major risks for reverse mortgages. If the borrower lives longer than expected, the principle advance and interest will continue to accumulate. It may cause the outstanding balance excess the proceeds from the sale of the property. Thus, the lenders of reverse mortgages are faced with longevity risk. A risk in interest rates can cause higher interest on the outstanding balance. Thus, it increases the risk that outstanding balance excesses the property value. If the property value decreases, it also increases the risk that outstanding balance excesses the proceeds from the sale of the property. That's called house price risk. In this research, we focus on the longevity risk and house price risk for reverse mortgages.

2.3 Modeling Possible Losses for HECM Program

Let H_t denote the property value and OB_t represent the loan balance at time t. If the loan is due at time t, the possible loss can be expressed as

$$L_t = \max\left(OB_t - H_t, 0\right) \quad , \quad \text{for } t = 1 \sim \omega - x \qquad (2\text{-}1)$$

where ω is maximal survival age.

We consider the loan is due only when the borrower dies in that year. Thus, the present value of expected total loss at time 0 is then calculated as

$$TL(0) = \sum_{t=1}^{\omega - x} E[e^{-rt} \, {}_{t-1}p_x \; q_{x+t-1} \cdot L_t] \qquad (2\text{-}2)$$

where ${}_{t-1}p_x$ denote the survival probability that the borrower aged x and survives to age x+t-1, q_{x+t-1} is the probability that the borrower dies in year t.

To transfer the longevity and house price risks for HECM program, we need to consider both mortality dynamic and house price dynamic, which are described below.

3. Modeling longevity risk

To examine the effect of longevity risk for reverse mortgages, we need a stochastic mortality model to capture future mortality dynamics properly. Various mortality models exist; for example, early developments of stochastic mortality modeling rely on the age-period effect and pioneering work by Lee and Carter (1992) and Renshaw and Haberman (2003) offer further analyses of the Lee-Carter model. Cairns et al. (2006a) instead consider their CBD model of functional relationships, which deals with mortality rates across ages and thus offers better performance for older persons. The CBD model also has been adapted to efforts to price longevity bonds or other mortality-linked securities. In line with the most recent literature on mortality modeling, we employ CBD stochastic mortality to model longevity risk for reverse mortgage and calculate the fair spread for the proposed security.

3.1 The CBD model

The CBD mortality model we used was proposed in Cairns, Blake, and Down (2006). They suggest a two-factor model for modeling initial mortality rates instead of central mortality rate. The mortality rate for a person aged x in year t ($q(t,x)$) is modeled as follows:

$$\text{logit } q(t,x) = k_t^1 + k_t^2 (x - \overline{x}) \tag{3-1}$$

where parameter k_t^1 represents the marginal effect with times on mortality rates and parameter k_t^2 portrays the old age effect on mortality rates and $\overline{x}$ is the mean age.

The future mortality can be projected

3.2 Parameter estimates

We estimate the parameters in the CBD model by fitting historical U.S. mortality data from 1950–2006 with the HMD data. The estimated parameters of k_t^1 and k_t^2 for males and females are depicted in Figure 1 and 2. k_t^1 shows a down trend and k_t^2 appear a upward trend. To project future mortality rates, we model k_t^1 and k_t^2 as follows.

$$\begin{bmatrix} \kappa_t^{(1)} \\ \kappa_t^{(2)} \end{bmatrix} = \begin{bmatrix} mu_1 \\ mu_2 \end{bmatrix} + \begin{bmatrix} \kappa_{t-1}^{(1)} \\ \kappa_{t-1}^{(2)} \end{bmatrix} + \begin{bmatrix} e_t^{(1)} \\ e_t^{(2)} \end{bmatrix} \tag{3-2}$$

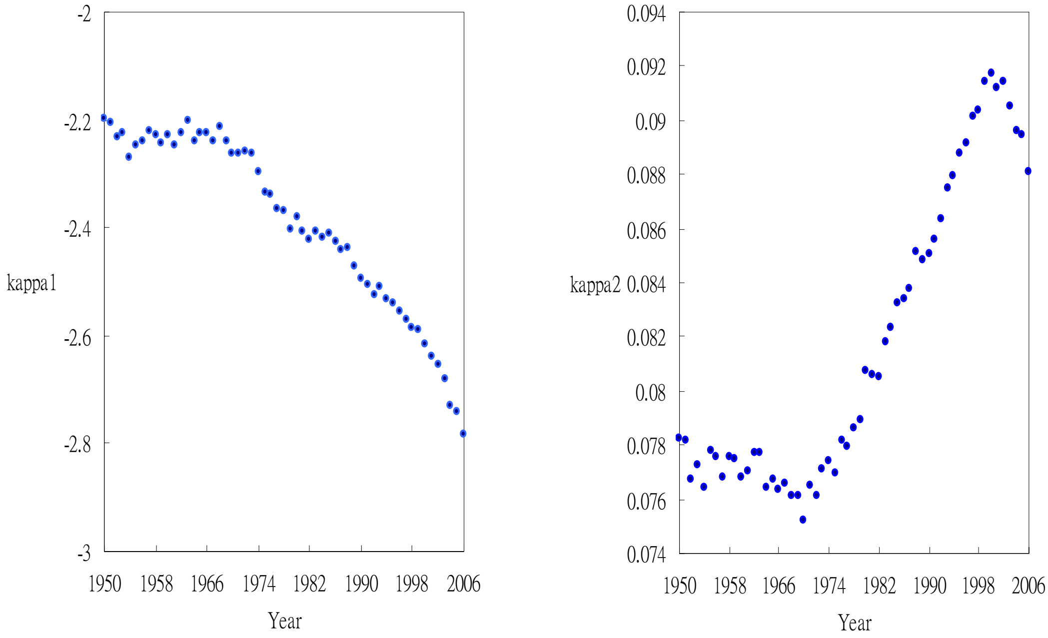

Figure 1. Estimated Values of kappa1(t) and kappa2(t) of Male, 1950 to 2006.

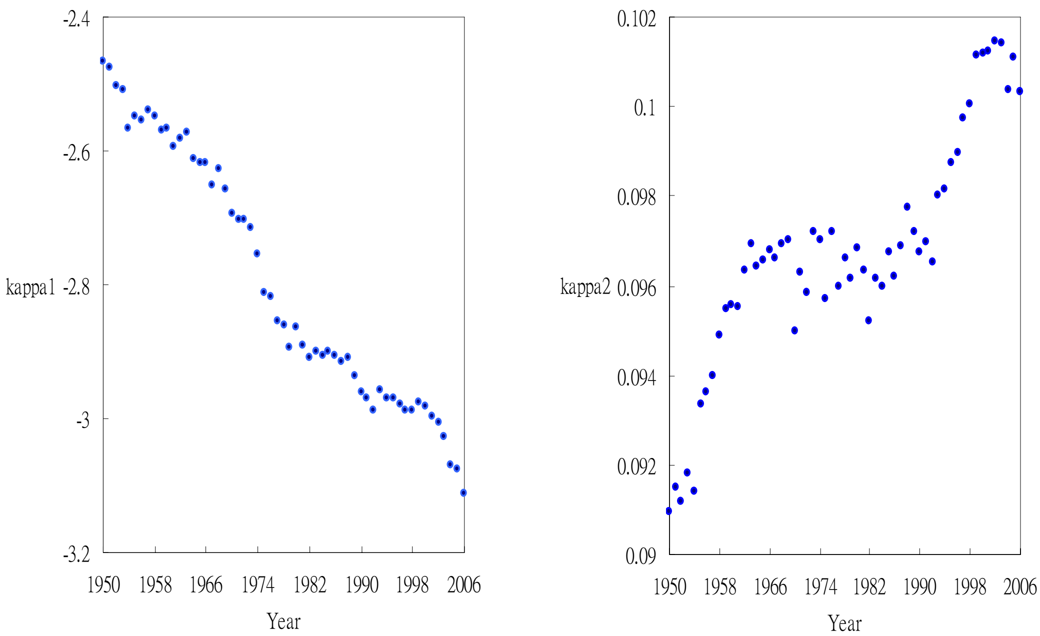

Figure 2. Estimated Values of kappa1(t) and kappa2(t) of Female, 1950 to 2006.

3.3 A Comparison of Projected Mortality Rates

Mortality modeling plays an important role in longevity securitization. We also examine the longevity risk modeled by the Lee-Carter model (Lee and Carter, 1992) to investigate the effect of model risk on securitization for reverse mortgages. The Lee-Carter (LC) model is emerging as a benchmark for mortality forecasts. To understand the distinction among the CBD, LC mortality models and static mortality rates, we compare the simulated survival probability for a man aged 62 years separately in Figure 3, which shows the survival probability projected by the CBD model is greater than that offered by the LC model. Since the static mortality rates ignoring the mortality improvement, the projected survival probability is much underestimated.

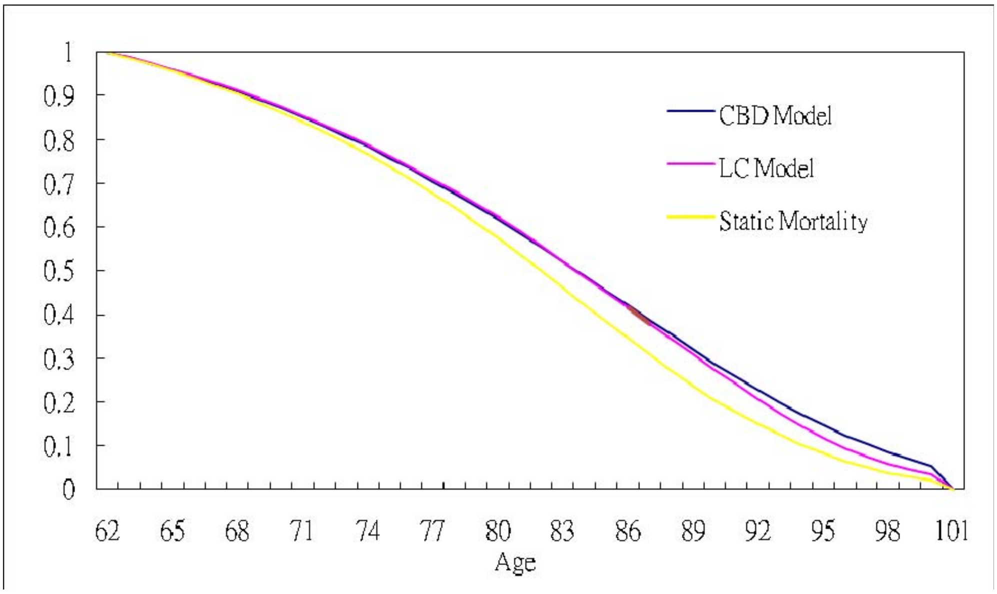

Figure 3. Simulated Survival Probability for American Males

4 Modeling Housing Price Dynamics

4.1 The House Price Data

Chen et al(2010) deal with HECM program and choose the nationwide house price index(HPI) to model the house price dynamics. We use the same house price data and extend the data period from the first quarter of 1975 to the first quarter of

2010. Let Y_t denote the log-return for house price index, which is defined as $Y_t = \log(H_t / H_{t-1})$. Based on the empirical HPI data(See Figure 4), the log- return of HPI is not stationary[5] and the first difference of log- return(DY_t) is stationary. Thus, we model the house price return based on the first difference of log- return, which is calculated as $Y_t - Y_{t-1}$.

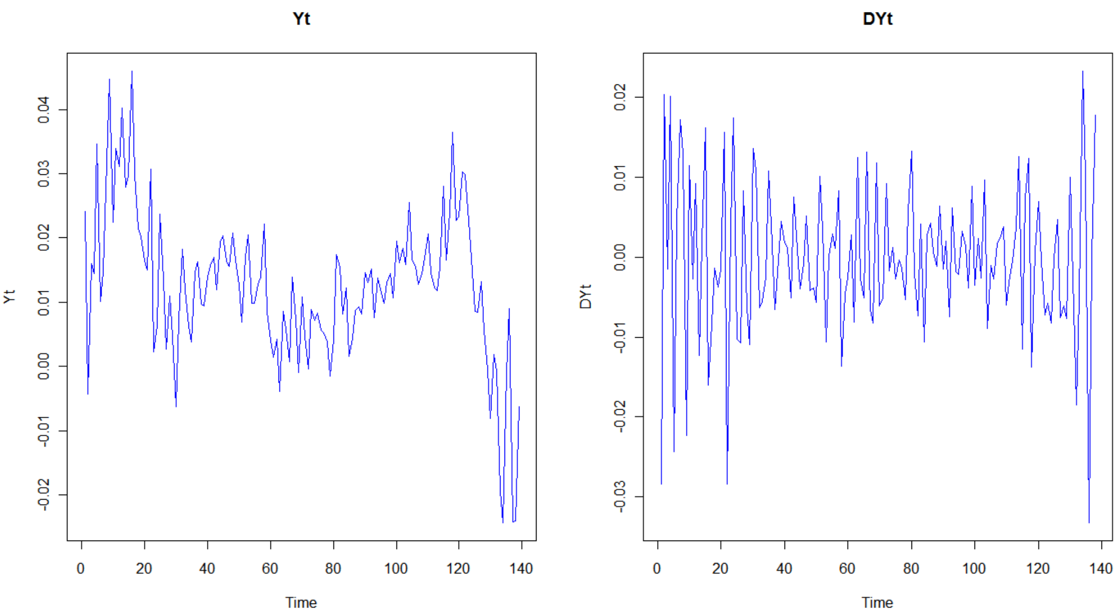

Figure 4 Log-return of HPI from 1975/Q1 to 2010/Q1

4.2 ARMA-GARCH Process

Li et al.(2010) point out three important properties for the house price dynamics in the U.K, which are autocorrelation, volatility clustering and leverage effects. Based on the nationwide HPI in the U.S., the leverage effect doesn't exist. Therefore, we consider the property of autocorrelation and volatility clustering to model the house price return dynamic and employ the ARMA-GARCH process. The model is expressed as

5 Both the ADF statistic and the PP statistic in CSXR are higher than the critical values at the significance level of 5%.

Under ARMA-GARCH model setting, the dynamics of house price return process is

$$DY_t = c + \sum_{i=1}^{m} \phi_i DY_{t-i} + \sum_{j=1}^{n} \theta_j z_{t-j} + z_t \quad (4\text{-}1)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i z_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (4\text{-}2)$$

4.3 Parameter Estimates

Based on the empirical data, we find the ARMA(2,0)-GARCH(1,1) model gives the best fit to the HPI data[6]. The parameter estimates are presented in Table 1.

Table 1 Parameter Estimates for ARMA(2,0)-GARCH(1,1) Process

Parameters	Estimate	Std. Error	t value	Pr(>|t|)
ϕ_1	-0.3830	0.0795	-4.8158	1.47E-06
ϕ_2	-0.4236	0.0820	-5.1638	2.42E-07
α_0	0.0000	2.47E-06	1.3988	0.1619
α_1	0.1045	0.0524	1.9965	0.0459
β_1	0.8288	0.0768	10.7853	0.00E+00

3. The Structure of Securitization for Reverse Mortgages and Valuation of CRMO

3.1 The Structure of Securitization for Reverse Mortgages

We propose a tranching security that the structure is similar to collateralized debt obligation(CDO). A CDO is an asset backed securitization where the underlying portfolio is comprised of securities (called a Collateralized Bond Obligation, CBO) or

[6] Chen et al.(2010) also find ARMA(2,0)-GARCH(1,1) gives the best fit to the data from 1975/Q1 to 2009/Q1.

loans (called a Collateralized Loan Obligation, CLO) or possibly a mixture of securities and loans. A CDO consists on tranching and selling the credit risk of the underlying portfolio. We design a securitization where the underlying portfolio is a pool of RM products, that is called collateralized reverse mortgage oblighation(CRMO) in this research. The structure of CRMO is described in Figure 1. The protection buyer, investment bank, pays the premiums (P) to the protection seller, special purpose vehicle (SPV). The SPV issues three tranches of the CRMO to investors with different degree of risk preferences. The total face amount of bonds issuing to investor is equal to (F). SPV invests the premium (P) and the proceeds from the sale of bonds (F) in default-free floating bonds with coupon rate (C_t)). If the loss (L_t) on the underlying RM product occurs, the tranche investor will receive the residual nominal value of contract($F - L_t$). Thus, the investor will bear the future uncertainty of loss and need to be compensated. The compensation return is called spread. In this research, we investigate the fair spreads for different tranche investors .

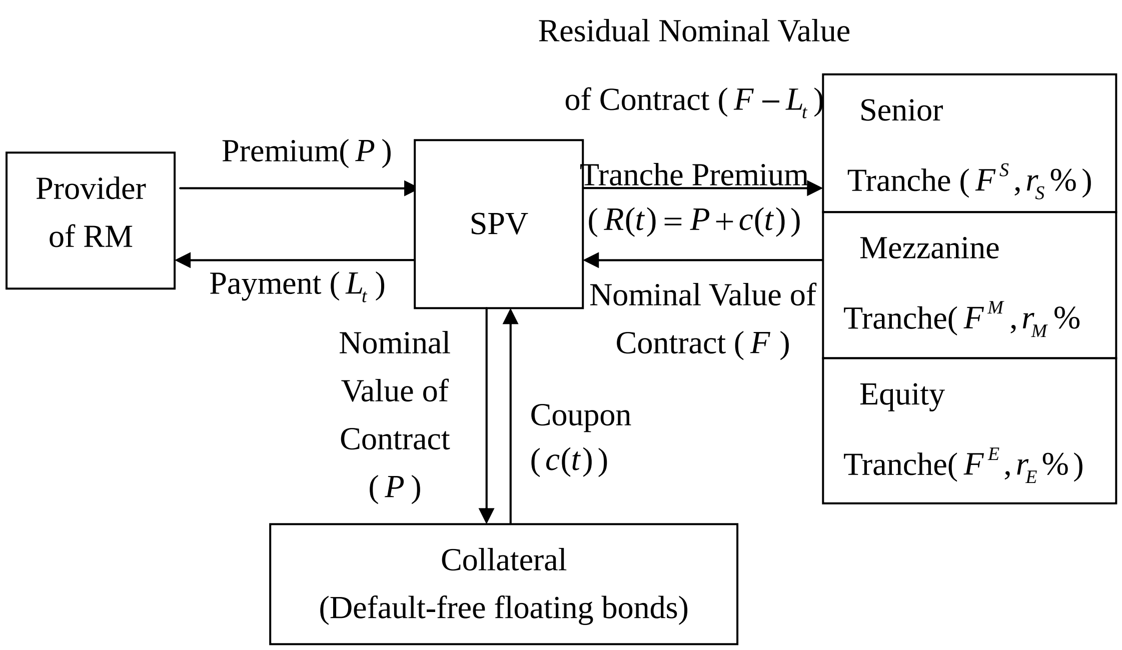

Figure 1 The Structure of CRMO

The SPV distributes the sales of bond to each tranche according to the tranching proportion($S\%, M\%,\ E\%$), that is $S\%$ of total face amount to Senior tranching investor, $M\%$ to Mezzanine tranche investor and the rest to the Equity tranche investor. The corresponding face amounts and fair spread for these three tranches are denoted as (F^S, F^M, F^E) and ($r_S\%, r_M\%,\ r_E\%$) separately.. Different tranche investors receive different spreads. At time t, when the loss occurs, the equity tranche first absorbs the loss and the face amount decreases the same amount. If the loss is larger than the residual face amount in the Equity tranche, the Mezzanine tranche has to be responsible for the loss. The Senior tranche is the last to absorbing the loss. The future residual face amount in Equity, Mezzanine and Senior tranches at time t can be expressed as

$$F_{t+1}^{E} = \begin{cases} F_t^E & \text{if } 0 \le L_t \\ F_t^E - L_t & \text{if } 0 < L_t \le F_t^E \\ 0 & \text{if } L_t > F_t^E \end{cases}$$

$$F_{t+1}^{M} = \begin{cases} F_t^M & \text{if } L_t \le F_t^{\mathrm{E}} \\ F_t^M - L_t & \text{if } F_t^{\mathrm{E}} < L_t \le F_t^M \\ 0 & \text{if } L_t > F_t^M \end{cases}$$

$$F_{t+1}^{\mathrm{S}} = \begin{cases} F_t^{\mathrm{S}} & \text{if } L_t \le F_t^M \\ F_t^{\mathrm{S}} - L_t & \text{if } F_t^M < L_t \le F_t^{\mathrm{S}} \\ 0 & \text{if } L_t > F_t^{\mathrm{S}} \end{cases}$$

For the investor’s point of view, the equity tranche first absorbs the loss. Thus, the investor’s in equity tranche is the most risky investor comparing with that for Mezzanine tranche and Senior tranche.

3.2 Valutaion of Collateralized reverse mortgage obligation(CRMO)

We propose a risk neutral valuation framework to find the fair spread for different tranche investors.. Based on the underlying portfolio of RM policies, the total losses are modeled according to Equation(2.1). Assume the loss occur at the middle year. At time 0, the present value of the expected total loss absorbing by Equity tranche investor is calculated as

$$L_E(0) = \sum_{t=1}^{T} e^{-r(t-0.5)} \left(E_Q[F_{t-1}^E - F_t^E] \right) \quad (3\text{-}1)$$

where r is the risk free rate, $\left(F_{t-1}^E - F_t^E\right)$ denotes the actual loss at time t that absorbed by Equity tranche and $E_Q[]$ represents the expectation under risk neutral measure .

The present value of the expected total compensation received by the Equity tranche investor is expressed as

$$P_E(0) = \sum_{t=1}^{T} e^{-rt} E_Q[r_E \%(F_{t-1}^E + F_t^E / 2)] \quad (3\text{-}2)$$

Thus, the fair spread(r_E) for Equity tranche investor can be obtained by setting

$$P_E(0) = L_E(0) \quad (3\text{-}3)$$

It implies that the expected total losses absorbed and the expected total compensation received by the Equity tranche investor shall be the same. The valuation formula applies to Mezzanine tranche and senior tranche. In addition, the present value of expected total loss presented in Equation(2-2) shall be equal to the total losses absorbed in different tranches, which is

$$TL(0) = L_E(0) + L_M(0) + L_S(0) \quad (3\text{-}4)$$

To calculate the risk neutral measure for house price dynamics and mortality dynamics, we employ the conditional Esscher transform to price the fair spreads for different tranche investors.

5. Numerical Illustration

5.1.Policy Setting for Reverse Mortgages

To illustrate the calculation of fair spread for the tranching security of RM products, we assume a portfolio of 1,000 identical RM policies issuing to the borrower aged 62 with initial property value of US$300,000 and loan taking in lump-sum payment. The total loan value is US$180,000,000. Assume the provider of RM want to transfer the entire risk. Thus, the SPV issues the bond with total face amount equal to US$180,000,000. The example of tranche level and the distribution of the face amount to each tranche are shown in Table 1. Based on the tranche level, we investigate the fair spread for different tranche investor.

Table 1 Illustration of Tranche Levels

	Level	Face Amount in Each Tranche
Senior tranche	5%	900,000
Mezzanine tranche	10%	1,800,000
Equity tranche	85%	15,300,000
Total	100%	180,000,000

Following the HECM program and the policy setting in Chen et al.(2010), we consider the transaction cost of selling the house(κ) and rental yield(g) in our numerical analysis. Assume $\kappa = 5\%$ and $g = 2\%$. The interest rate charged on the loan(i) is assumed to be 2.42%. The mortality model and the house price model used to model the possible losses are presented in Section 3 and 4. The risk free rate is assumed to be constant and we use 3.78%. We carry out 100,000 Monte Carlo Simulations to calculate the numerical results.

4.2. Shortfalls Analysis for Issuing RM Products

We first analyze the shortfalls for the provider when issuing a portfolio RM Products. Based on different loan to values, Table 2 presents the shortfalls in terms of value at risk(VaR) and conditional tail expectation(CTE). As expected, the higher the LTV, the higher the shortfalls. For the LTV equal to 90% , the probability of no loss occurring is less than 1%(0.059% actually) and CTE(90%) is US$200,753 for females. Comparing the results with that of LTV equal to 50%, the probability of no loss occurring is around 9.14% and, CTE(90%) is US$109,429 for females. The shortfall is much smaller. Thus, the LTV is very critical to the risk involved in issuing RM policy. Transferring the risk for RM provider is necessary especially for higher LTV.

Table 2 Shortfall for a Portfolio of RM Policies

LTV	Gender	Prob. of no Loss (Loss=0)	max	mean	VaR 90%	VaR 95%	VaR 99%	CTE 90%	CTE 95%	CTE 99%
50%	Male	9.1420%	117,348	67,203	102,257	105,004	109,123	105,540	107,524	110,724
	Female	9.1420%	115,747	67,821	101,444	104,016	107,951	104,511	106,368	109,429
60%	Male	6.1320%	141,360	85,692	124,788	127,741	132,226	128,324	130,468	133,941
	Female	6.1320%	138,938	85,434	123,008	125,833	130,227	126,415	128,481	131,916
70%	Male	3.4570%	165,676	105,276	147,759	150,867	155,619	151,503	153,780	157,490
	Female	3.4570%	162,348	103,848	144,897	147,943	152,737	148,593	150,855	154,640
80%	Male	1.2920%	190,171	125,813	171,121	174,396	179,336	175,038	177,421	181,324
	Female	1.2920%	185,864	122,940	167,065	170,311	175,459	171,017	173,466	177,578
90%	Male	0.0590%	214,857	147,158	194,887	198,274	203,442	198,949	201,433	205,519
	Female	0.0590%	209,829	142,593	189,457	192,934	198,446	193,689	196,314	200,753

4.3. Analysis of Fair Spreads

The fair spreads of CRMO for different tranches are presented in Table 3. For a illustration purpose, we calculate the fair spreads for both male and female borrowers and different LTV of 50%, 60%, 70%, 80% and 90%. Based on the LTV of 60%, the fair spread for Equity tranche is 8.517%, for Mezzanine is 4.7445% and for Senior is

1.5507% for male borrowers. Different tranche investors bear different degree of risk. Since the equity tranche investor absorbs the loss first than other tranche investor, the fair spread in equity tranche is much higher than other tranches. The fair spreads for the three tranches are lower for female borrowers. This is because the life expectancy for female is longer.

We further compare the fair spreads for the underlying reverse mortgage portfolio with different LTV. As expected the higher the LTV, the higher the fair spread is. Since the higher LVT may result in higher loss, the investor needs to bear more risk and requires more compensation. In addition, the change in spreads is more significant for the Equity tranche investors and less significant for Senior tranche investors. For the LVT of 50% to 90%, the fair spread changes from 6.8348% to 16.6392% for Equity tranche investors, more than 9.5% difference; but from 1.1106% to 2.5513% for Senior tranche investors, less than 1.5% difference.

Table 3 Fair Spreads for Different Tranches of CRMO ($\lambda = 1\%$)

LTV	RM Borrower	Equity	Mezzanine	Senior
		5%	10%	85%
50%	Male	6.8348%	3.9543%	1.1106%
	Female	6.0213%	3.4357%	1.1083%
60%	**Male**	**8.5170%**	**4.7445%**	**1.5507%**
	Female	**7.3288%**	**4.0291%**	**1.4789%**
70%	Male	10.5874%	5.6100%	1.9178%
	Female	8.8611%	4.6538%	1.7535%
80%	Male	13.2030%	6.5736%	2.2405%
	Female	10.6897%	5.3208%	1.9941%
90%	Male	16.6392%	7.6562%	2.5513%
	Female	12.9189%	6.0372%	2.2222%

In table 4, the fair spreads are calculated assuming the risk premium is 0.1. We investigate the risk premium assumption on the calculation of fair spread. Table 3 presents the fair spread based on risk premium assumption of 0, 0.1, 0.2 and 0.3. As expected, the higher the risk premium in pricing, the lower the fair spread is. Howver, the effect is not significant.

Table 4 The Effect of Risk Premium on Fair Spreads ($LTV = 60\%$)

Risk Premium		Equity	Mezzanine	Senior
	RM Borrower	5%	10%	85%
$\lambda = 0.0$	Male	8.5379%	4.7742%	1.5878%
	Female	7.3518%	4.0611%	1.5222%
$\lambda = 0.1$	**Male**	**8.5170%**	**4.7445%**	**1.5507%**
	Female	**7.3288%**	**4.0291%**	**1.4789%**
$\lambda = 0.2$	Male	8.4746%	4.6839%	1.4626%
	Female	7.2823%	3.9637%	1.3731%
$\lambda = 0.3$	Male	8.4315%	4.6214%	1.3561%
	Female	7.2349%	3.8960%	1.2402%

Securitization longevity risk for annuity business has been widely discussed. The discussion of longevity risk to reverse mortgages is still under development. The earlier HECM program uses static mortality tables to calculate the loan value. To investigate the effect of failing to capture the dynamics of mortality on securitization of longevity risk for reverse mortgages, Table 5 presents the result using static mortality table[7]. Comparing the results with Table 3, the fair spread increases in each tranche. From the SPV point of view, ignoring mortality dynamic will overestimate the fair spread.

Table 5 The Effect of Ignoring Mortality dynamics on Fair Spreads for Different Tranches of CRMO

LTV		Equity	Mezzanine	Senior
	RM	5%	10%	85%

[7] Based on HMD, we use the mortality experience in year 2006 as the static mortality table.

	Borrower			
50%	Male	7.0406%	4.1889%	1.2092%
	Female	6.2948%	3.6770%	1.2733%
60%	**Male**	**8.7726%**	**5.0098%**	**1.7013%**
	Female	**7.6850%**	**4.3140%**	**1.6627%**
70%	Male	10.9134%	5.9085%	2.1138%
	Female	9.3307%	4.9906%	1.9550%
80%	Male	13.6423%	6.9111%	2.4731%
	Female	11.3257%	5.7170%	2.2175%
90%	Male	17.2569%	8.0450%	2.8118%
	Female	13.8042%	6.5044%	2.4662%

Finally, we investigate the fair spread based on different mortality model. We employ the LC model to calculate the fair spread. In most of the cases, the fair spread based on LC model is lower. This is because the projected life expectancy using CBD model is a little bit longer than that using LC model.

Table 5 The Effect of Mortality Model Risk on Fair Spreads for Different Tranches of CRMO($\lambda = 1\%$): LC model

LTV		Equity	Mezzanine	Senior
	RM Borrower	5%	10%	85%
50%	Male	6.7880%	3.9369%	1.1960%
	Female	5.9314%	3.3354%	1.1964%
60%	Male	8.4363%	4.7091%	1.6363%
	Female	7.2463%	3.9101%	1.5297%
70%	Male	10.4537%	5.5537%	1.9799%
	Female	8.8061%	4.5256%	1.7723%
80%	Male	12.9877%	6.4899%	2.2874%
	Female	10.6971%	5.1920%	1.9911%
90%	Male	16.3010%	7.5381%	2.5848%
	Female	13.0548%	5.9191%	2.2007%

5. Conclusion

In recent years, reverse mortgages are getting more popular in many countries. To transfer the risk inherent in reverse mortgage products, we propose a securitization method. The proposed securitization structure differs from existing literature by introducing the tranche longevity and house price risks for reverse mortgages. The structure of securitization for reverse mortgages is similar to that for collateralized debt obligation (CDO). Different to price CDO, we model the dynamics of future mortality and house price instead of default rate. Thus, we model the house price index using ARMA-GARCH process. To deal with longevity risk for elders, we use the CBD model (Cairns et al, 2006) to project future mortality. We propose a risk neutral valuation framework and employ the conditional Esscher transform to price the fair spreads for different tranche investors. The problems of using static mortality table and model risk on pricing fair spread are investigated numerically.

In our numerical analysis, we first analyze the shortfalls for the provider when issuing a portfolio RM Products. We calculate the fair spreads for both male and female borrowers and different LTV of 50%, 60%, 70%, 80% and 90%. Since the equity tranche investor absorbs the loss first than other tranche investor, the fair spread in equity tranche is much higher than other tranches. The fair spreads for the three tranches are lower for female borrowers. This is because the life expectancy for female is longer. We further compare the fair spreads for the underlying reverse mortgage portfolio with different LTV. As expected the higher the LTV, the higher the fair spread is. Since the higher LVT may result in higher loss, the investor needs to bear more risk and requires more compensation. In addition, the change in spreads is more significant for the Equity tranche investors and less significant for Senior tranche investors.

The earlier HECM program uses static mortality tables to calculate the loan value. We also investigate the effect of failing to capture the dynamics of mortality on securitization of longevity risk for reverse mortgages. From the SPV point of view, ignoring mortality dynamic will overestimate the fair spread. In addition, we employ the LC model to calculate the fair spread. In most of the cases, the fair spread based on LC model is lower. This is because the projected life expectancy using CBD model is a little bit longer than that using LC model. Thus, it implies the use of LC model will underestimate the fair spread to the investors.

In the light of our analysis in this paper, we also highlight some areas for further research. First, we do not investigate the issue of jump effect with house price dynamic and mortality rates. Second, we ignore stochastic interest rate in the valuation framework. Third, we only consider the reverse mortgages in the form of lump-sum payment. These three areas would be valuable and interesting extensions for additional research to tackle.

Reference

1. Cairns, Andrew J.G., David Blake and Kevin Dowd (2006). A Two-factor Model for Stochastic Mortality with Parameter Uncertainty: Theory and Calibration, *Journal of Risk and Insurance,* 73: 687-718.
2. Chen, Hua, Samuel H. Cox, Shaun S. Wang(2010).Is the Home Equity Conversion Mortgage in the United States sustainable? Evidence from pricing mortgage insurance premiums and non-recourse provisions using the conditional Esscher transform. Insurance: Mathematics and Economics 46:, 173-185.
3. David Blake, Anja De Waegenaere, Richard MacMinn, Theo Nijman, 2009, Longevity Risk and Capital Markets: The 2008-2009 Update.
4. Lee, R. D. and L.R. Carter (1992). Modeling and Forecasting U. S. Mortality, *Journal of the American Statistical Association,* 87: 659-671.
5. Li, J. S. H., M. R. Hardy and K. S. Tan(2010). On Pricing and Hedging the No-Negative-Equity Guarantee in Equity Release Mechanisms, Journal of Risk and Insurance, 77: 449-522.
6. Liao, H.H., S.S. Yang and I.H. Huang(2007). The Design of Securitization for Longevity Risk: Pricing under Stochastic Mortality Model with Tranche Technique. Presented at the Third International Longevity Risk and Capital Market Solutions Symposium, Taipei.
7. Wills, S, and M. Sherris(2010). Securitization, Structuring and Pricing of Longevity risk, Insurance: Mathematics and Economics, 46 (1), 173-185.
8. Sherris, M. and S. Wills (2007). Financial innovation and the hedging of longevity risk. Asia Pacific Journal of Risk and Insurance 3(1), 52-64.
9. Zhai, David H.(2000), Reverse Mortgage Securitizations: Understanding and Gauging the Risks, Special Report, Moody’s Investors Service.